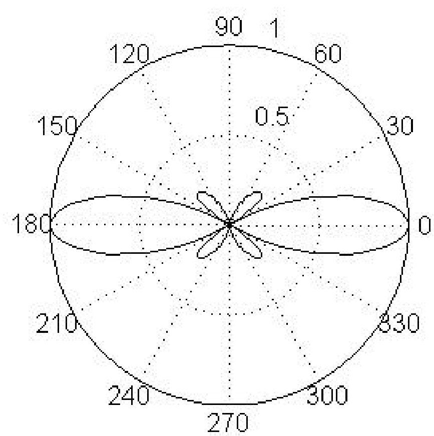
Antennas & Propagation

LECTURE NOTES
VOLUME III

ARRAYS, ANTENNAS IN SYSTEMS AND ACTIVE ANTENNAS

by Professor David Jenn

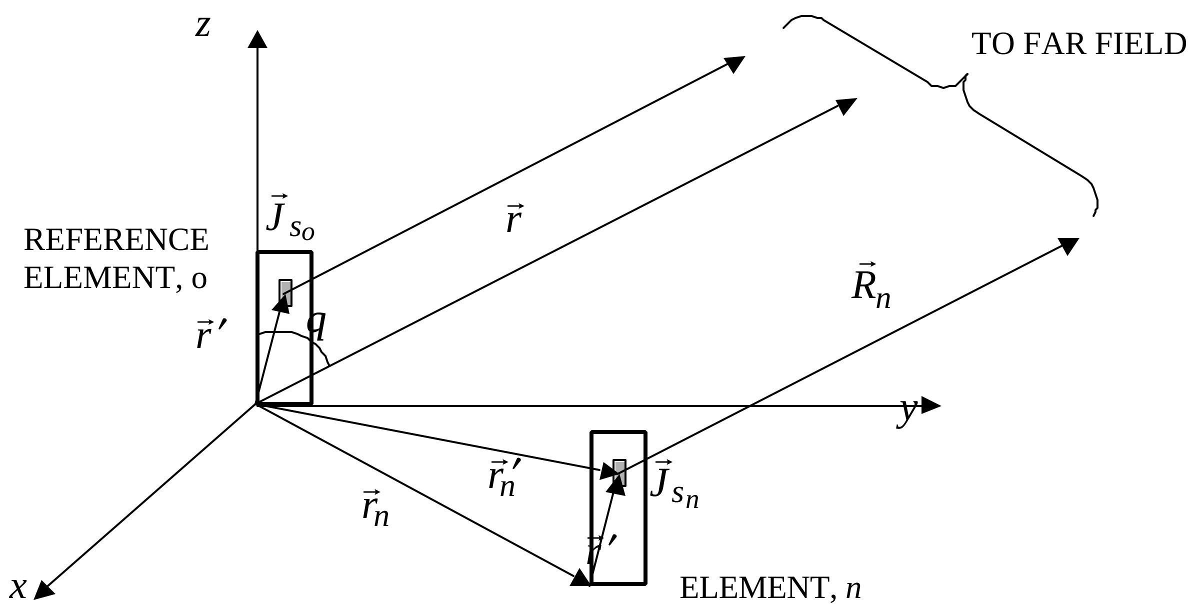


(ver 1.3)

Array Antennas (1)

Arrays are collections of antennas that are geometrically arranged and excited in such a way as to achieve a desired radiation pattern. In most applications the array elements are identical (e.g., an array of dipoles). The general geometry of an array is shown below.

- The observation point is in the far field
- The global coordinate system is located at the reference element (#0)
- The location of element *n* is given by the position vector $\vec{r}_n$



Array Antennas (2)

The radiation integral for the *n*th element is

$$\vec{E}(r,q,f) = \frac{-jkh}{4p\, r} e^{-jkr} \iint_{S_n} \vec{J}_{s_n} e^{jk\vec{r}_n{}' \cdot \hat{r}} ds'$$

The total field at P is the sum of all of the radiated element fields. Assume that the current <u>distribution</u> on all elements is the same except for a complex constant scaling factor, $A_n = a_n e^{j\Phi_n}$ $(a_n \equiv |A_n|)$:

$$\vec{J}_{s_n} = A_n \vec{J}_{s_o}$$

Noting that $\vec{r}_n{}' = \vec{r}\,' + \vec{r}_n$, the total field is

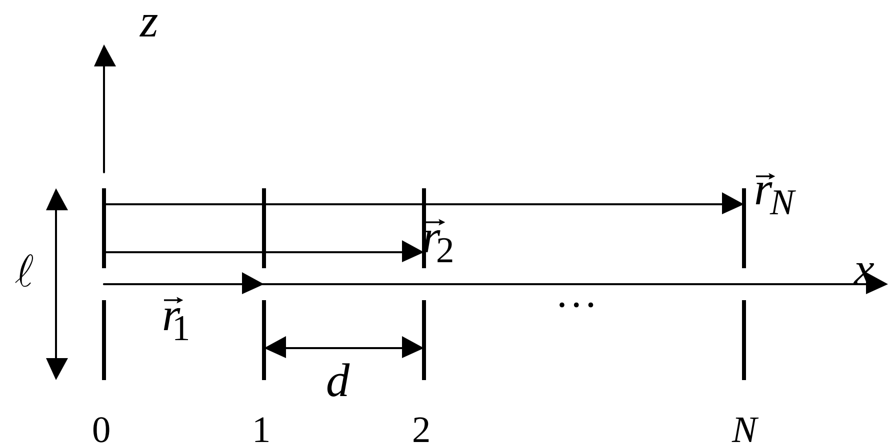
$$\vec{E}(r,q,f) = \sum_{n=0}^{N} \left(\frac{-jkh}{4p\, r_n} \right) e^{-jkr} \iint_{S_n} A_n \vec{J}_{s_o} e^{jk(\vec{r}\,' + \vec{r}_n)\cdot \hat{r}} ds'$$

$$= \underbrace{\left[\frac{-jkh}{4p\, r} e^{-jkr} \iint_{S_n} \vec{J}_{s_o} e^{jk\vec{r}\,' \cdot \hat{r}} ds' \right]}_{\text{ELEMENT FACTOR,EF}} \underbrace{\left[\sum_{n=0}^{N} A_n e^{jk\vec{r}_n \cdot \hat{r}} \right]}_{\text{ARRAY FACTOR,AF}}$$

This result is referred to as the <u>principle of pattern multiplication</u>.

Array Antennas (3)

As a simple application of the array formulas, consider a linear array of dipoles along the *x* axis.



Assume: 1. equally spaced dipole elements
2. uniformly excited (equal power to all elements, $A_n = 1$)
4. neglect "edge effects" (mutual coupling changes near edges)

From the coordinate tables $\hat{r} = \hat{x} \sin q \cos f + \hat{y} \sin q \sin f + \hat{z} \cos q$. The position vector to element n is $\vec{r}_n = nd\,\hat{x}$. Therefore

$$\hat{r} \bullet \vec{r}_n = nd \sin q \cos f$$

Array Antennas (4)

Now we can write the array factor

$$\mathrm{AF} = \sum_{n=0}^{N} A_n e^{jk\vec{r}_n \cdot \hat{r}} = \sum_{n=0}^{N} (1) e^{jknd \sin q \cos f} = \underbrace{\sum_{n=0}^{N} \left(e^{jy}\right)^n}_{\text{GEOMETRIC SERIES}} = \frac{1 - \left(e^{jy}\right)^{N+1}}{1 - e^{jy}}$$

where $y = kd \sin q \cos f$. Using trigonometric identities

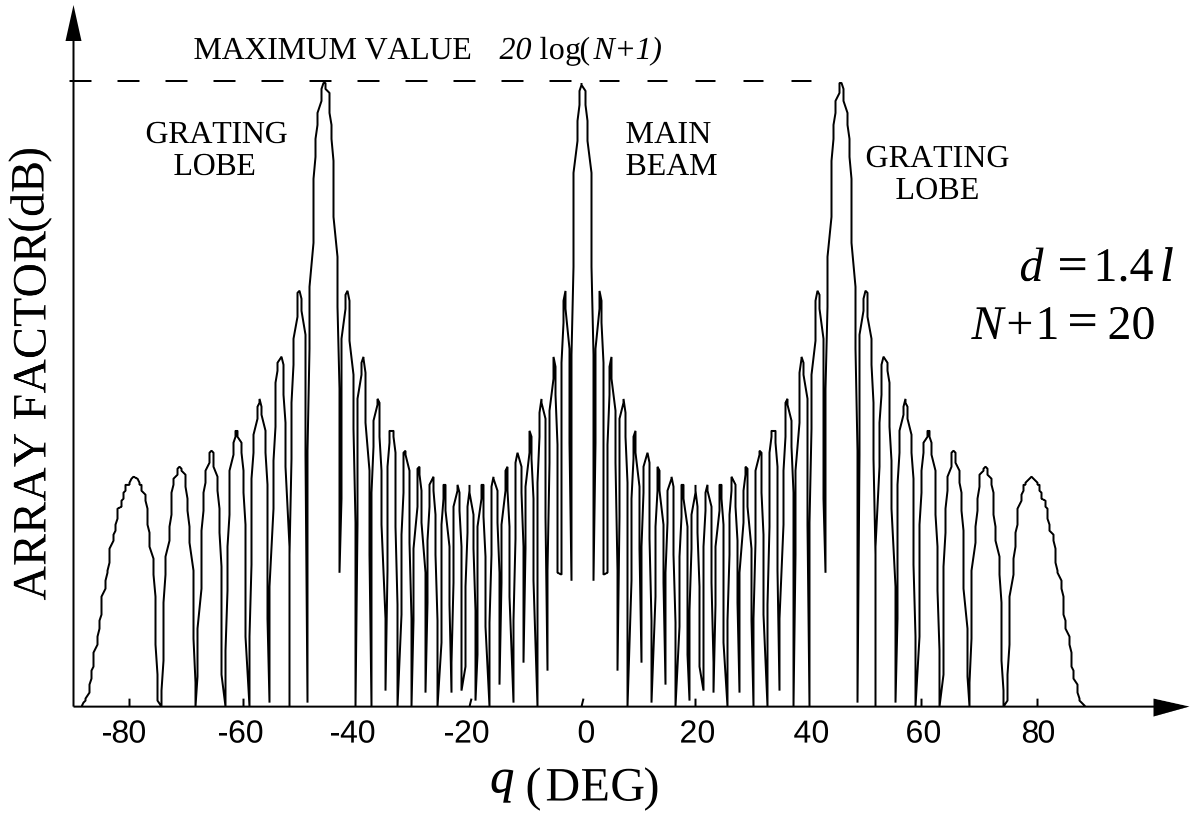
$$\mathrm{AF} = \frac{\sin[(N+1)y/2]}{\sin(y/2)} e^{jNy/2} \quad \Rightarrow \quad |\mathrm{AF}| = \left| \frac{\sin[(N+1)y/2]}{\sin(y/2)} \right|$$

The maximum value of AF is N+1 and it occurs whenever the denominator is zero

$$\sin(y/2) = 0 \quad \Rightarrow \quad y_m/2 = mp \quad \Rightarrow \quad y_m = 2mp, \quad m = 0, \pm 1, \ldots$$

A plot of the array factor is shown on the next page. The maximum at $y = 0$ is the main beam or main lobe. The other maxima are called grating lobes. Generally grating lobes are undesirable, because most applications such as radar require a single focussed beam. There are N-1 sidelobes between each grating lobe; the notches between lobes are nulls.

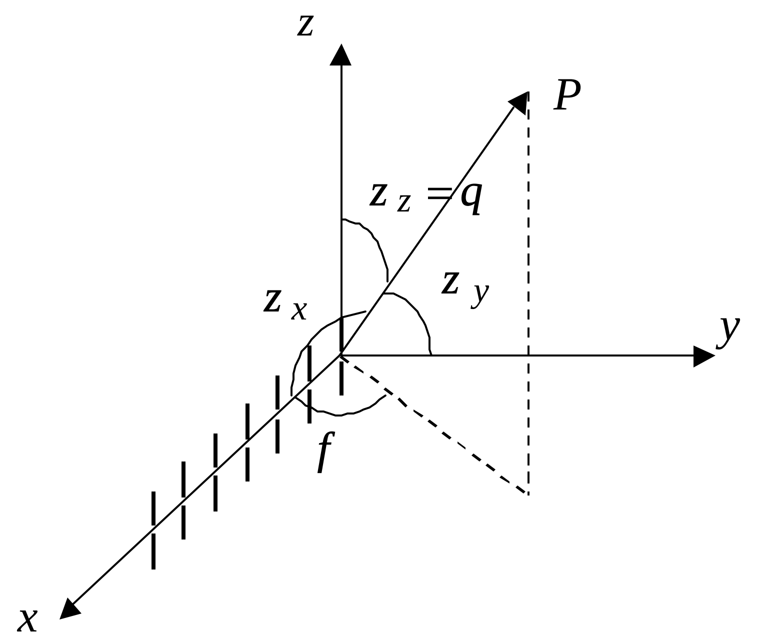
Array Antennas (5)



The array factor is defined for $-\infty < y < \infty$. The region that corresponds to the spatial window of $-90^\circ < q < 90^\circ$ is called the <u>visible region</u>.

Array Antennas (6)

We have considered an array along the *x* axis, as shown. To obtain the array factor for arrays on other axes, simply replace the *x* direction cosine with the direction cosine of the appropriate axis.



Direction cosines:

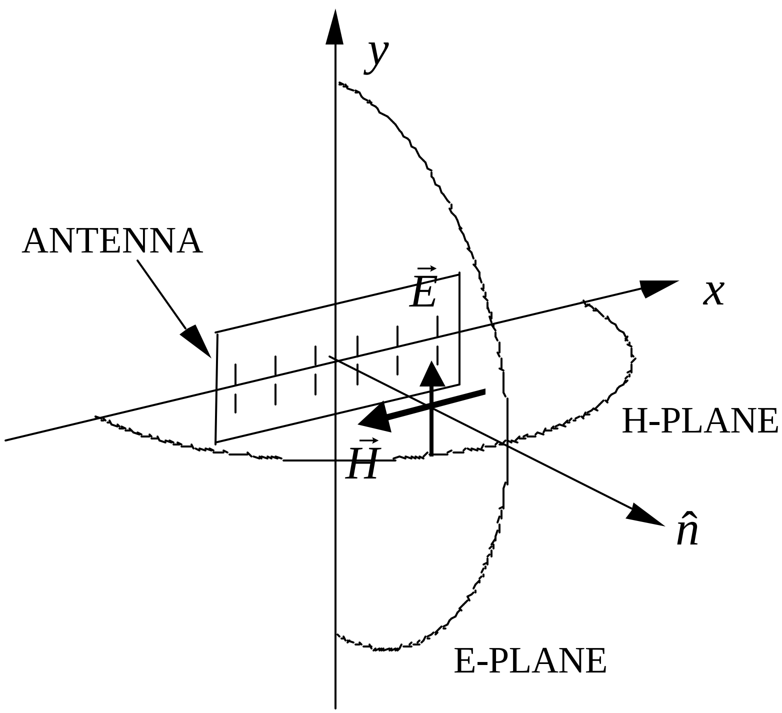
$$\cos z_x = \sin q \cos f = u$$
$$\cos z_y = \sin q \sin f = v$$
$$\cos z_z = \cos q = w$$

For example, if the array is located along the *y* axis use $y = kd\underbrace{\sin q \sin f}_{v}$

Principal Planes

The principal planes of a linearly polarized antenna are defined by looking directly at the antenna (along the normal $\hat{n}$) from the far field. If the electric and magnetic fields in the aperture plane are TEM, then $\vec{E}$ and $\vec{H}$ are orthogonal. The two principal planes are

- E-plane: defined by the vectors $\hat{n}$ and $\vec{E}$
- H-plane: defined by the vectors $\hat{n}$ and $\vec{H}$

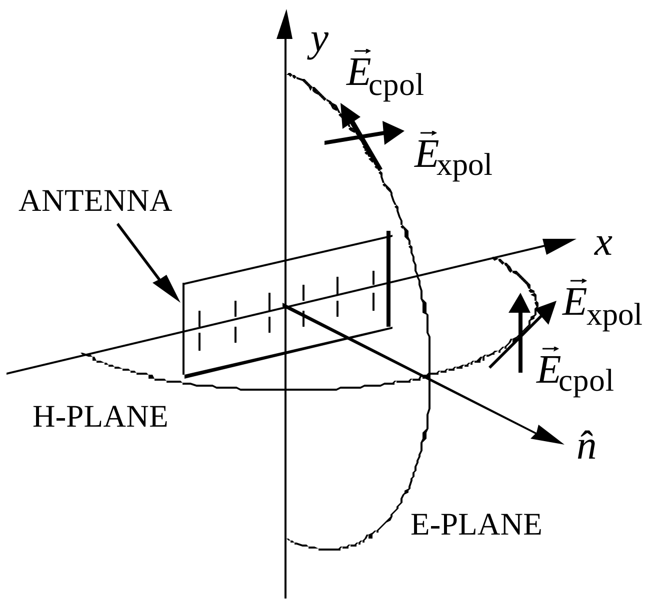


For example, if an array of dipoles are aligned with the *y* axis, then the *y*-*z* plane is the E plane ($\vec{E}$ is parallel to the dipoles). The *x*-*z* plane is the H plane.

Polarization Reference

In a principal plane, the component of $\vec{E}$ parallel to the field in the aperture (i.e., lies in the principal plane and is parallel to the effective height vector, $\vec{h}_e$) is the co-polarized component. The component of $\vec{E}$ perpendicular to the field in the aperture is the cross polarized component.

- in the E-plane: $E_{\mathrm{cpol}} = E_q$ and $E_{\mathrm{xpol}} = E_f$
- in the H-plane: $E_{\mathrm{cpol}} = E_f$ and $E_{\mathrm{xpol}} = E_q$

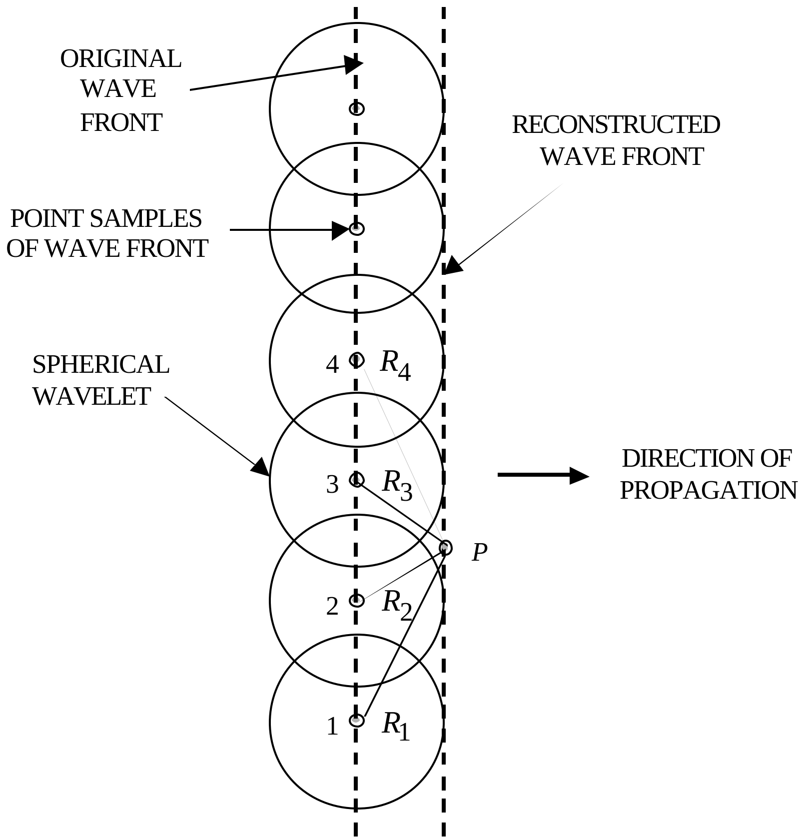


The crossed polarized radiation for well-designed array antennas is very low. The maximum crossed polarized value is typically 40 to 60 dB below the maximum value of the co-polarized component.

When an array of crossed dipoles operate as two orthogonal linear arrays, the two sets of dipoles can operate simultaneously with high isolation between the two polarizations. This is referred to as polarization diversity (or polarization reuse).

Huygen's Principle (1)

Huygen's principle states that any wavefront can be decomposed into a collection of point sources. New wavefronts can be constructed from the combined "spherical wavelets" from the point sources of the old wavefront.

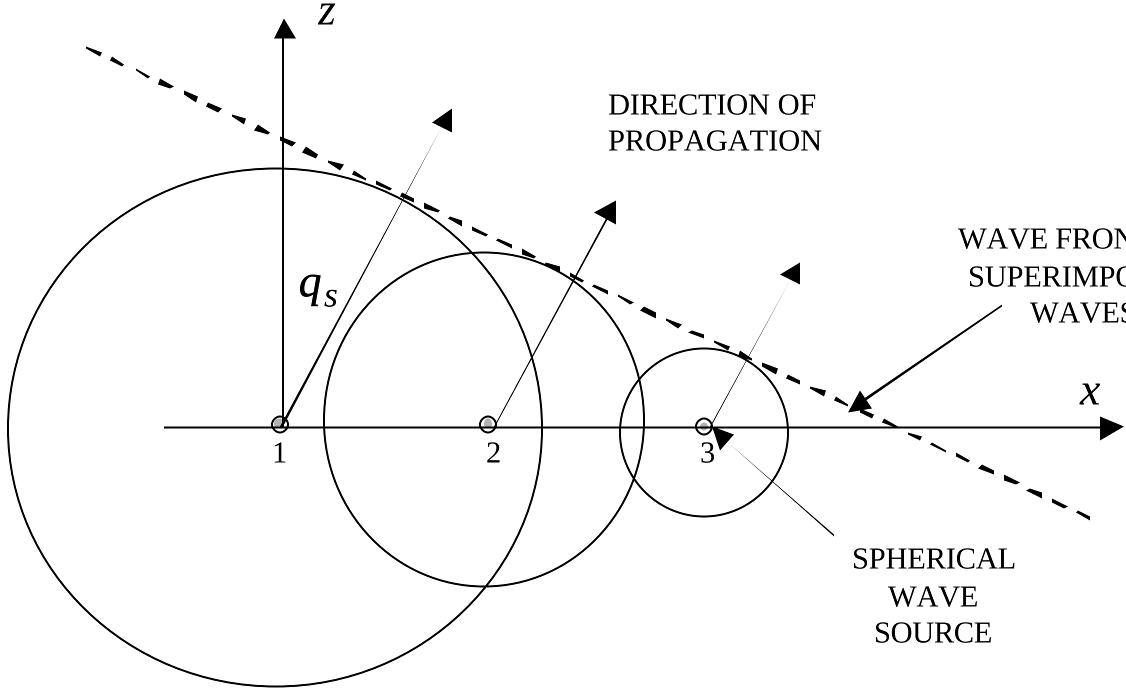


$$E(P) \sim \sum_{n=-\infty}^{\infty} \frac{e^{-jkR_n}}{R_n}$$

where R_n is the distance from the nth source to the observation point, P. Sources closest to P will contribute most to the field

Huygen's Principle (2)

An array of sources can be used to generate wavefronts in any desired direction. This is easily illustrated in the time domain. Assume that source 1 is turned on first, then source 2, and finally source 3. A plane wave approximation will be generated, and the angle of propagation depends on the time delay increment between sources.

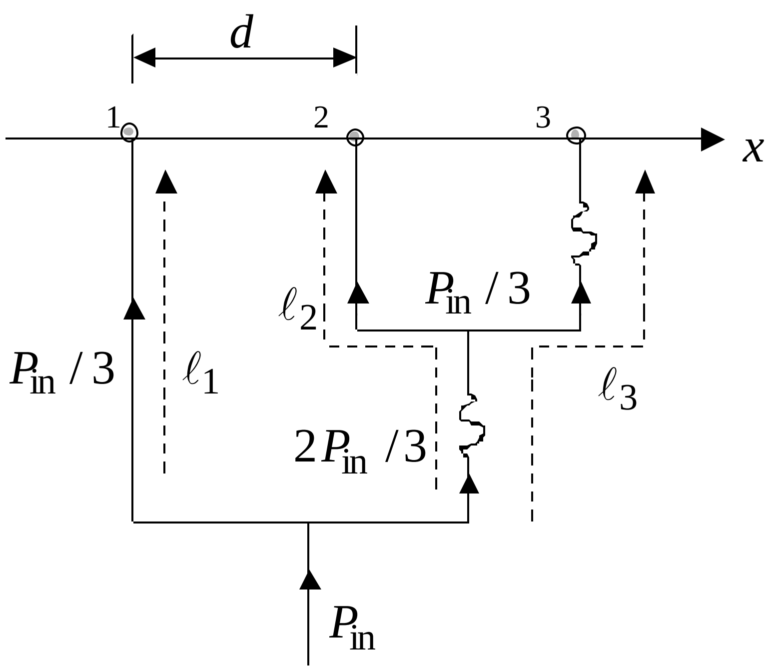


Since time-harmonic waves are sinusoidal in time and space, the same result occurs if a phase delay is introduced between adjacent sources.

Huygen’s Principle (3)

Time-harmonic waves vary as $\sim \cos(wt - kz)$ where z is the direction of propagation. Increasing z has the same effect as delaying time.

Parallel feed arrangement:



- Line lengths for time delay:

  ℓ_1 is arbitrary

  $$\ell_2 = \ell_1 + d \sin q_s$$

  $$\ell_3 = \ell_1 + 2d \sin q_s$$

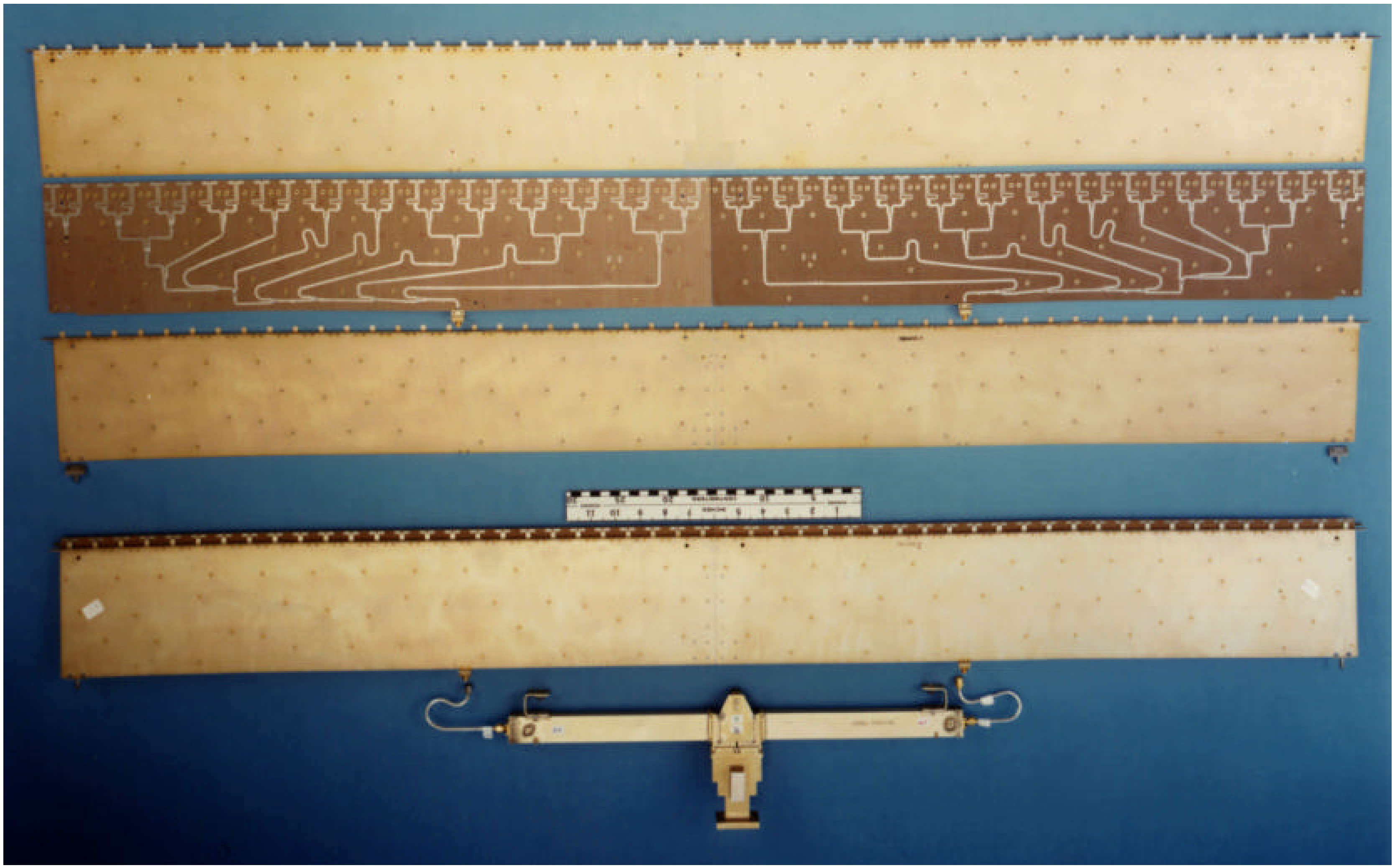
- The associated phases are

  $$k\ell_1, k\ell_2 \text{ and } k\ell_3$$

- In phasor form $e^{-jk\ell_1}$, $e^{-jk\ell_2}$ and $e^{-jk\ell_3}$
  - After subtracting out the common phase factor, the residual is

  $$e^0,\ e^{-jkd \sin q_s} \text{ and } e^{-j2kd \sin q_s}$$

Printed Circuit Dipole Array



Scanned Arrays (1)

An appealing feature of arrays is that the pattern can be scanned electronically by adjusting the array coefficients, A_n. This approach permits beams to be moved from point to point in space in just microseconds, whereas mechanical scanning of a large antenna could take several seconds. Let the excitation coefficients be an imaginary quantity (a "phase shift") that has a constant linear progression from element to element

$$A_n = e^{-jnkd \sin q_s \cos f_s} = e^{-jny_s}$$

where $y_s = kd \sin q_s \cos f_s$ and (q_s, f_s) is the direction in which the beam is to be pointed. The array factor becomes

$$\mathrm{AF} = \sum_{n=0}^{N} e^{jknd(\sin q \cos f - \sin q_s \cos f_s)} = \sum_{n=0}^{N} e^{jn(y - y_s)} = \frac{\sin[(N+1)(y - y_s)/2]}{\sin[(y - y_s)/2]} e^{jN(y - y_s)/2}$$

The magnitude of the array factor is

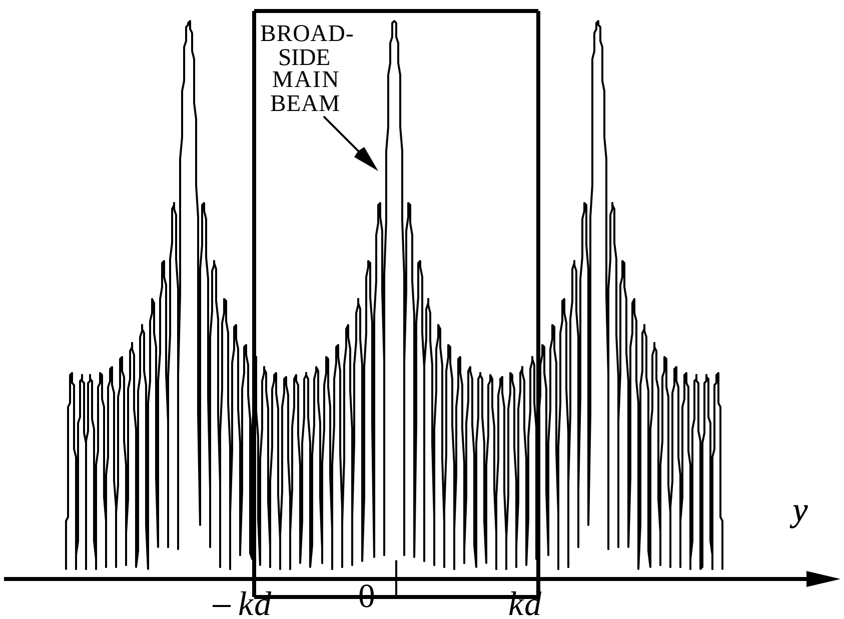
$$|\mathrm{AF}| = \left| \frac{\sin[(N+1)(y - y_s)/2]}{\sin[(y - y_s)/2]} \right|$$

The main beam is now located in the direction $y = y_s$. The array factor slides through the visible region window until the main beam is located at y_s.

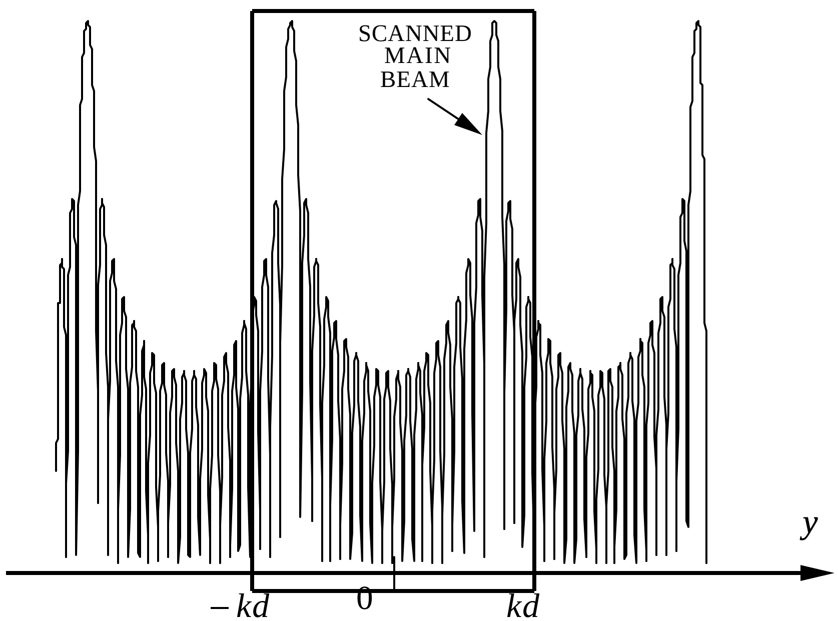
Scanned Arrays (2)

The location of the array factor in the visible window is shown for the cases of no scan and with scan. Notice that grating lobes originally outside of the visible region when the beam is not scanned can enter the visible region when the beam is scanned.

Broadside beam (no scan)



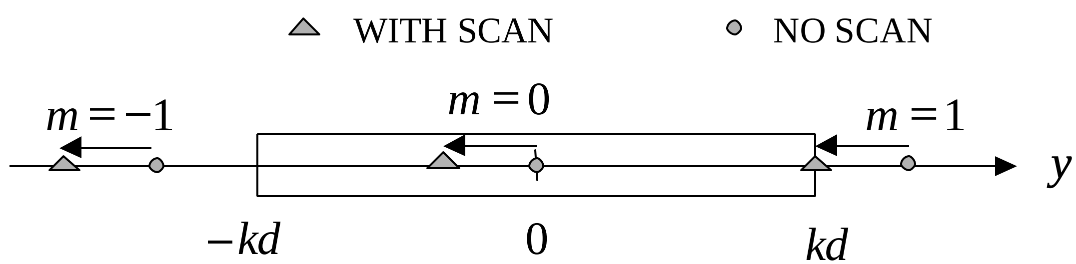
Scanned beam



Grating Lobe Example (1)

Example: A linear array of elements $d = 0.6 l$. At what scan angle will the first grating lobe appear at the edge of the visible region?

The result will be the same whether the beam can be scanned in either the positive or negative direction. The first grating lobe on the positive side is $m = 1$, and it will enter the visible region when the beam is scanned in the negative direction. For the given spacing, $kd = \frac{2p}{l} 0.6 l = 1.2p$. The grating lobe and main beam locations are depicted below.



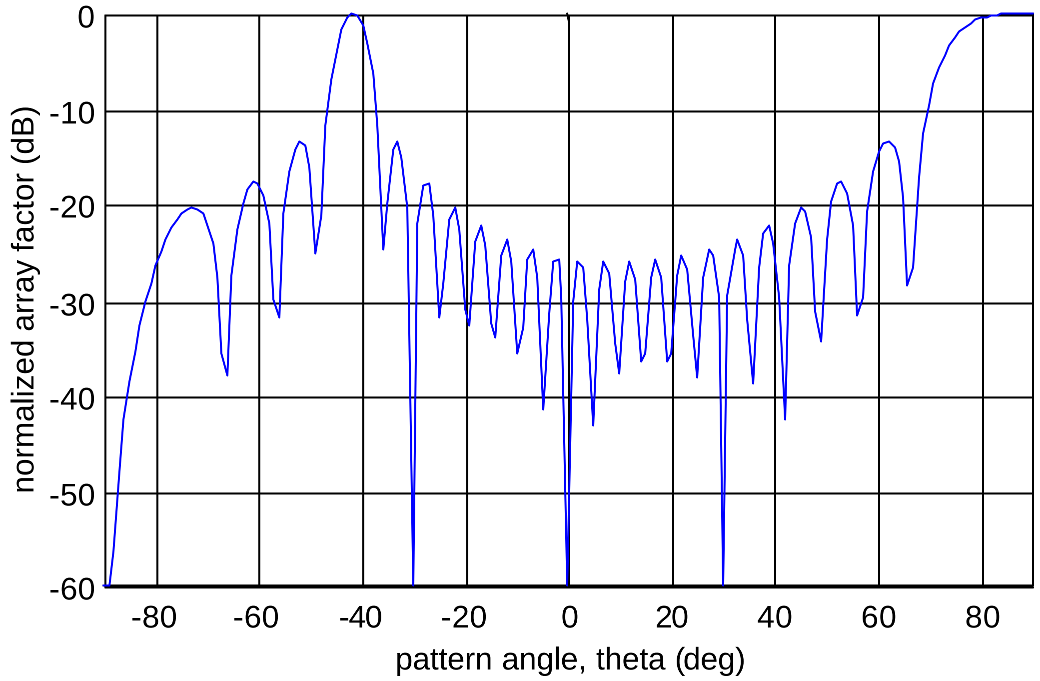
Working with the $m = 1$ lobe, which enters at $q = 90^\circ$,

$$\frac{y - y_s}{2} = \frac{kd\left(\sin 90^\circ - \sin q_s\right)}{2} = p$$

$$\sin q_s = 1 - l / d = -0.667 \quad \Rightarrow \quad q_s = -41.8^\circ$$

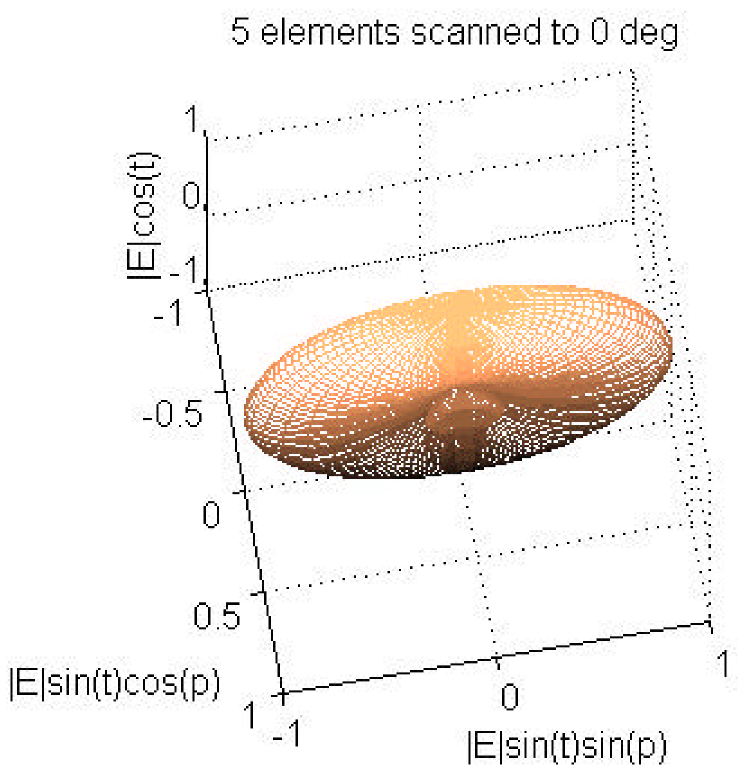
Grating Lobe Example (2)

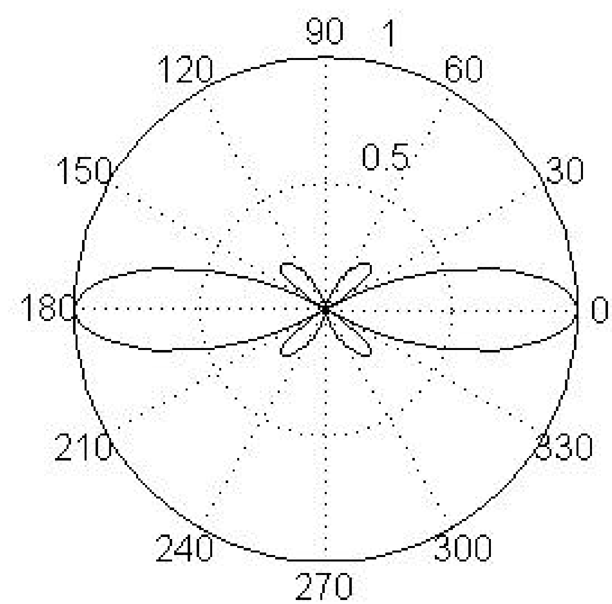
Array factor plot for a 20 element array that is scanned to –41.8 degrees. The element spacing is $d = 0.6l$. Note that the grating lobe location does not depend on the number of elements, but only the spacing in wavelengths. The total pattern is obtained by multiplying by EF, which could be chosen to place a null at the same location as the grating lobe. Note how the main beam broadens when it is scanned away from broadside.



Sample Linear Array Patterns (1)

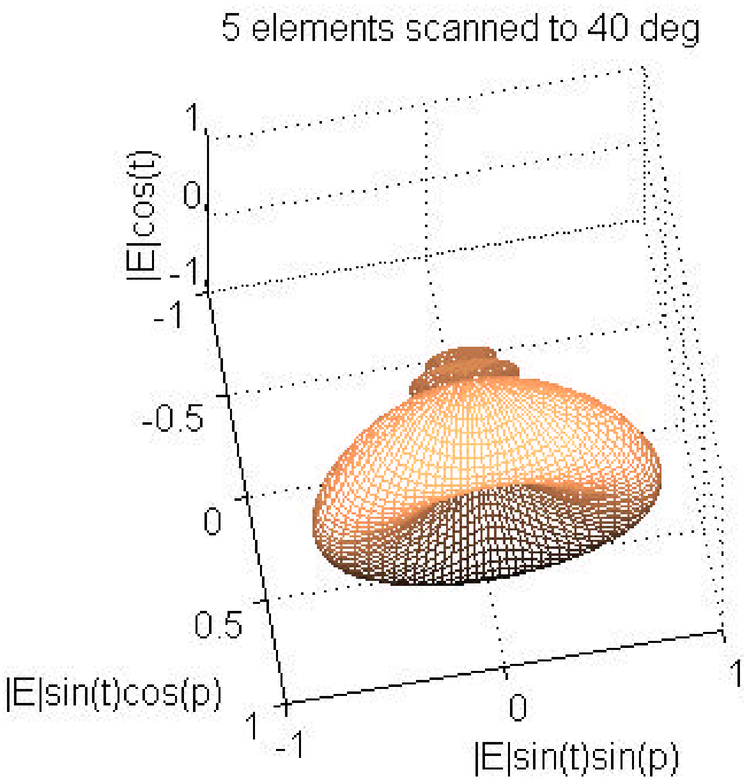
AF for 5 element array, $d = 0.4 l$

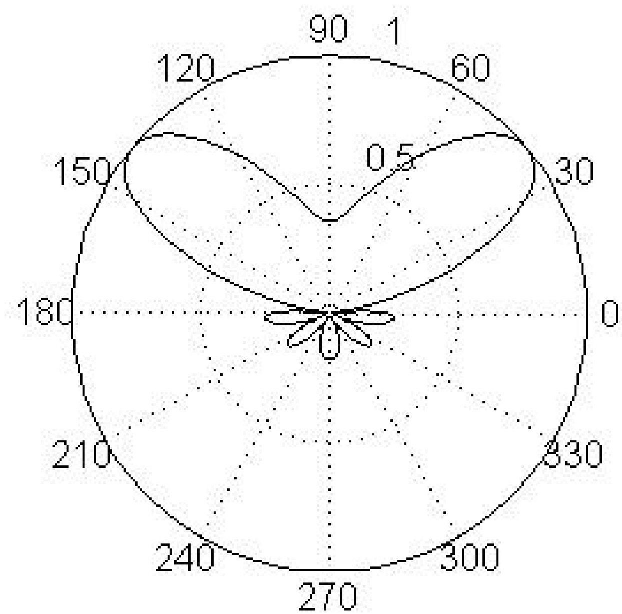




Sample Linear Array Patterns (2)

AF for 5 element array, $d = 0.4 l$





Array Beamwidth (1)

The main lobe width is commonly described by either the 3 dB, half-power beamwidth, (HPBW) or the beamwidth between first nulls (BWFN).

The beamwidth between first nulls can be determined from the numerator of the array factor,

$$\sin\left[\frac{N+1}{2}\left(y_1 - y_s\right)\right] = 0 \quad \Rightarrow \quad \frac{N+1}{2}\left(y_1 - y_s\right) = \pm p$$

where $y_1 = kd\sin q_1$. Assume that the scan angle is zero, and that the array length is approximately $L \approx (N+1)d$

$$\frac{L}{2}\frac{2p}{l}\sin q_1 = \pm p$$

$$\sin q_1 = \pm\frac{l}{L}$$

$$q_1 = \pm\sin^{-1}\left(\frac{l}{L}\right)$$

Therefore, for a symmetrical beam, $\mathrm{BWFN} = 2\left|q_1\right| = 2\sin^{-1}\left(\frac{l}{L}\right)$

Array Beamwidth (2)

If the array is very long in terms of wavelength, then L/λ is large and the small angle approximation is valid

$$\text{BWFN} \approx \frac{2\lambda}{L}$$

More commonly the HPWB is used. Since power is proportional to field squared:

$$|\text{AF}_{\text{norm}}|^2 = \left| \frac{\sin[(N+1)\psi_{HP}/2]}{(N+1)\sin(\psi_{HP}/2)} \right|^2 = 0.5$$

where $\psi_{HP} = kd\sin\theta_{HP}$ and θ_{HP} is the half power angle. Generally this must be done on the computer or by interpolating from a table. Once θ_{HP} is found, the half power beamwidth is

$$\text{HPBW},\ \theta_B = 2\theta_{HP}$$

This assumes that the beam is symmetrical about the peak. If the beam is asymmetrical, which occurs when it is scanned off of broadside, the half power points on the left and right sides of the beam are not equal. Each one must be computed separately and then added to obtain the HPBW.

Linear Array Directivity (1)

From the definition of directivity,

$$D_o = \frac{4\pi}{\Omega_A}$$

$$\Omega_A = \int_0^{2\pi}\int_0^{\pi} \left|\vec{E}_{\mathrm{norm}}(\theta,\phi)\right|^2 \sin\theta \, d\theta \, d\phi$$

Assume a uniform array. For convenience align the array along the *z* axis. Then the *z* direction cosine is used rather than the *x* direction cosine in the argument of AF

$$|\mathrm{AF}| = \left|\frac{\sin[(N+1)(\psi - \psi_s)/2]}{\sin[(\psi - \psi_s)/2]}\right|$$

where $\psi = kd\cos\theta$ and $\psi_s = kd\cos\theta_s$. If the beam is not scanned far from broadside, then the pattern will not deviate significantly from the non-scanned case. Therefore let $\psi_s = 0$. Now evaluate the integral

$$\Omega_A = \int_0^{2\pi}\int_0^{\pi} \left|\frac{\sin[(N+1)\psi/2]}{(N+1)\sin[\psi/2]}\right|^2 \sin\theta \, d\theta \, d\phi$$

Linear Array Directivity (2)

There is an identity that can be used if the beam is near broadside:

$$\left|\mathrm{AF}_{\mathrm{norm}}\right|^2 = \frac{1}{N+1} + \frac{2}{(N+1)^2}\sum_{n=0}^{N}(N+1-n)\cos(nkd\cos\theta)$$

Inserting this in the integrand and evaluating the integral gives

$$\Omega_A = 4\pi\left\{\frac{1}{N+1} + \frac{2}{(N+1)^2}\sum_{n=0}^{N}\left[\frac{(N+1-n)}{nkd}\sin(nkd)\right]\right\}$$

Finally,

$$D_o = \frac{N+1}{1+\dfrac{1}{kd}\displaystyle\sum_{n=1}^{N}\left[\frac{(N+1-n)}{(N+1)n}\sin(nkd)\right]}$$

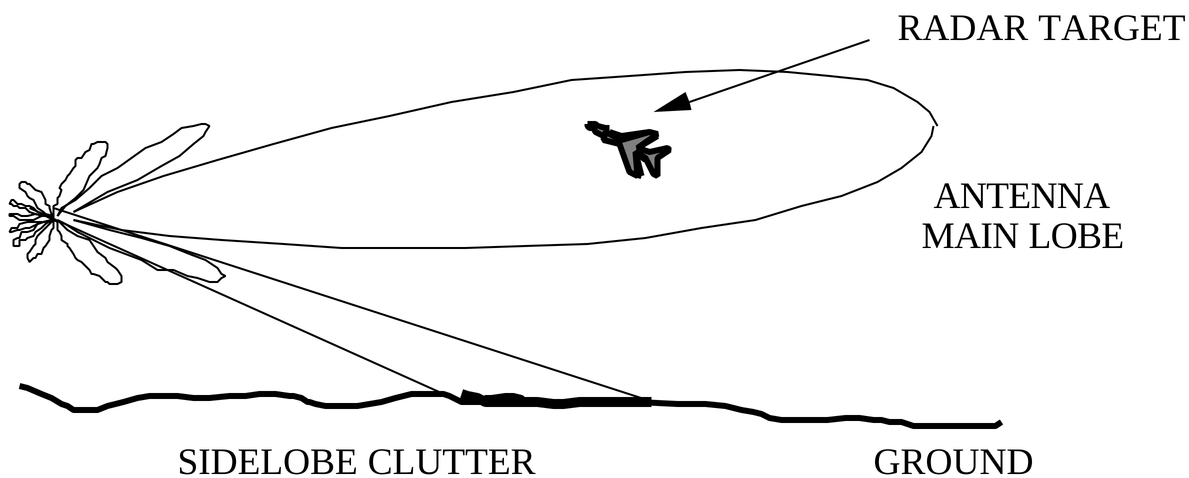
Example: N elements with $d = \lambda/2;\ kd = \pi \quad \rightarrow \quad \sin(nkd) = 0$ and

$$D_o = N+1$$

Antenna Sidelobe Control (1)

Many antenna applications require low sidelobes and scanned beams. Some of the important advantages and disadvantages of sidelobe reduction are:

Advantages	Disadvantages
reduced clutter return	more complicated feed required
low probability of intercept (LPI)	reduced gain
less susceptible to jamming	increased beamwidth; nulls move outward

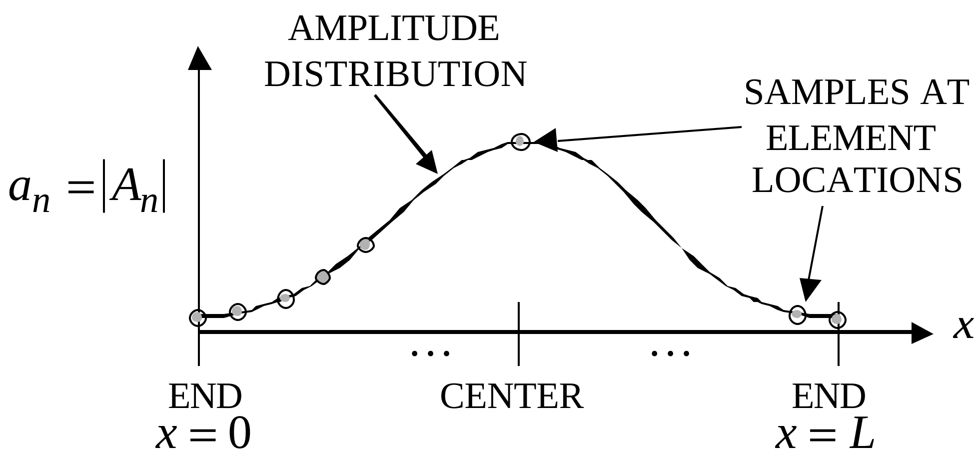


Classification of sidelobe level (not standardized):

- low sidelobes: -25 to – 40 relative to the beam maximum
- ultra-low sidelobes: < – 40 relative to the beam maximum

Antenna Sidelobe Control (2)

The shape of the amplitude distribution, determined by $a_n = |A_n|$, can be used to reduce the sidelobes of the radiation pattern. For a focused beam the amplitude distribution is always symmetric about the center of the array. To scan a focused beam a linear phase is introduced across the array length.

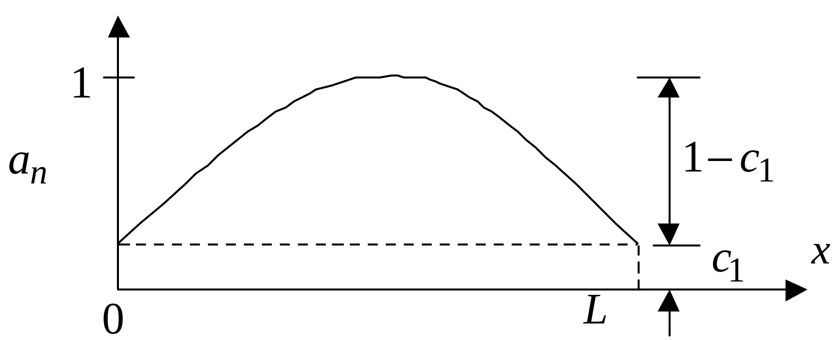


Common aperture distribution functions:

1. Chebyshev:
    - yields the minimum beamwidth for a specified sidelobe level
    - all sidelobes are equal
    - only practical for a small number of elements

Antenna Sidelobe Control (3)

2. Binomial:
   - has no sidelobes
   - only practical for a small number of elements
3. Taylor:
   - specify the maximum sidelobe level and rate of falloff of sidelobe level
4. Bayliss:
   - for low sidelobe difference beams (a difference beam has a null where the pattern would normally have a maximum)
5. Cosine-on-a-pedestal: (cosine raised to a power plus a constant)
   - wide range of sidelobe levels and falloff rates
   - Hamming window is one of these



$$a_n = c_1 + (1-c_1)\cos^m\left(\frac{p\,n}{2N}\right)$$

Aperture Efficiency

The reduction in directivity due to non-uniform amplitude is given by the illumination efficiency

$$e_i = \frac{\left|\sum_{n=0}^{N} a_n\right|^2}{(N+1)\sum_{n=0}^{N}\left|a_n\right|^2}$$

For an array this factor is sometimes called the aperture efficiency. However, for other types of antennas such as reflectors, the aperture and illumination efficiencies are not necessarily the same.

Example: For a uniform distribution, $a_n = 1$, and therefore

$$e_i = \frac{\left|\sum_{n=0}^{N} a_n\right|^2}{(N+1)\sum_{n=0}^{N}\left|a_n\right|^2} = \frac{\left|\sum_{n=0}^{N} 1\right|^2}{(N+1)\sum_{n=0}^{N}\left|1\right|^2} = \frac{(N+1)^2}{(N+1)(N+1)} = 1$$

For a cosine distribution, $a_n = \cos\left(\frac{p x_n}{L}\right)$, where x_n is the location of element *n*. The efficiency can be found with a simple computer calculation. Choose an arbitrary value for *N*, generate the coefficients, and then calculate e_i. The result is $e_i \approx 0.81 = -0.91$ dB.

Some Common Amplitude Distributions

The table summarizes the characteristics of some commonly encountered amplitude distributions. It is taken from *Introduction to Radar Systems* by Skolnik, (3rd edition) but similar tables can be found in any antenna textbook. The values are usually computed for continuous distributions. We can use the numbers for arrays (which can be viewed as sampled versions of continuous distributions) if the elements are closely spaced enough so that the true shape of the distribution function is approximated.

Table 9.1 in Skolnik

- z is the distance along the array (x in previous charts)
- a is the array length (L in previous charts)
- Relative gain is e_i

λ = wavelength; a = aperture width

Type of distribution, $\lvert z \rvert < 1$	Relative gain	Half-power beamwidth, deg	Intensity of first sidelobe, dB below maximum intensity
Uniform; $A(z) = 1$	1	$51\lambda/a$	13.2
Cosine; $A(z) = \cos^n(\pi z/2)$:			
$n = 0$	1	$51\lambda/a$	13.2
$n = 1$	0.810	$69\lambda/a$	23
$n = 2$	0.667	$83\lambda/a$	32
$n = 3$	0.575	$95\lambda/a$	40
$n = 4$	0.515	$111\lambda/a$	48
Parabolic; $A(z) = 1 - (1 - \Delta)z^2$:			
$\Delta = 1.0$	1	$51\lambda/a$	13.2
$\Delta = 0.8$	0.994	$53\lambda/a$	15.8
$\Delta = 0.5$	0.970	$56\lambda/a$	17.1
$\Delta = 0$	0.833	$66\lambda/a$	20.6
Triangular; $A(z) = 1 - \lvert z \rvert$	0.75	$73\lambda/a$	26.4
Circular; $A(z) = \sqrt{1 - z^2}$	0.865	$58.5\lambda/a$	17.6
Cosine-squared plus pedestal;			
$0.33 + 0.66 \cos^2(\pi z/2)$	0.88	$63\lambda/a$	25.7
$0.08 + 0.92 \cos^2(\pi z/2)$, Hamming	0.74	$76.5\lambda/a$	42.8

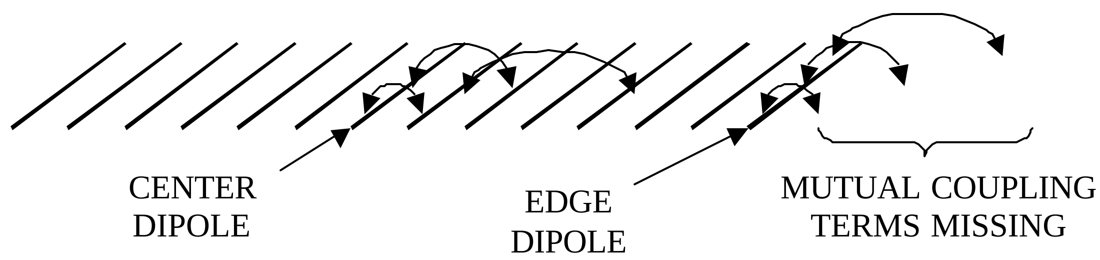
Summary of Array Characteristics

1. The radiation pattern of an array of identical elements is the product of an element factor and an array factor. This implies that the current distribution on every element is the same except for a constant scale factor, A_n. This assumption is not accurate for small arrays, or for the elements of large arrays that are located near edges.
2. Grating lobes (secondary maxima) occur when the spacing relative to wavelength becomes too large. Grating lobes originally outside of the visible region can move into the visible region when the beam is scanned.
3. The directivity increases with array length if there are no grating lobes. This implies that more elements must be added to increase the length.
4. The HPBW (and BWFN) decrease with increasing array length.
5. Scanning can be achieved by providing a linear phase progression per element.
6. The HPBW (and BWFN) increase as the beam is scanned from broadside. The beam shape between the half power points (or first nulls) becomes asymmetrical when scanned.
7. The sidelobe levels can be reduced by symmetrically tapering the excitation amplitude from the center to the edges.
8. A reduction in sidelobe level is accompanied by an increase in beamwidth and a reduction in gain.

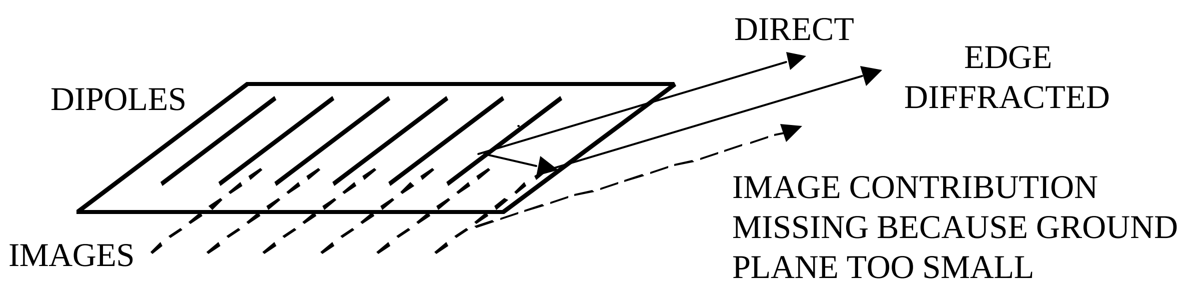
Finite Arrays and the "Edge Effect" (1)

The patterns of finite arrays differ from those of infinite arrays. The difference is referred to as the edge effect.

- Mutual coupling variations are significant near the edges of the array. Elements near the edges have fewer neighbors than those in the center of a large array.

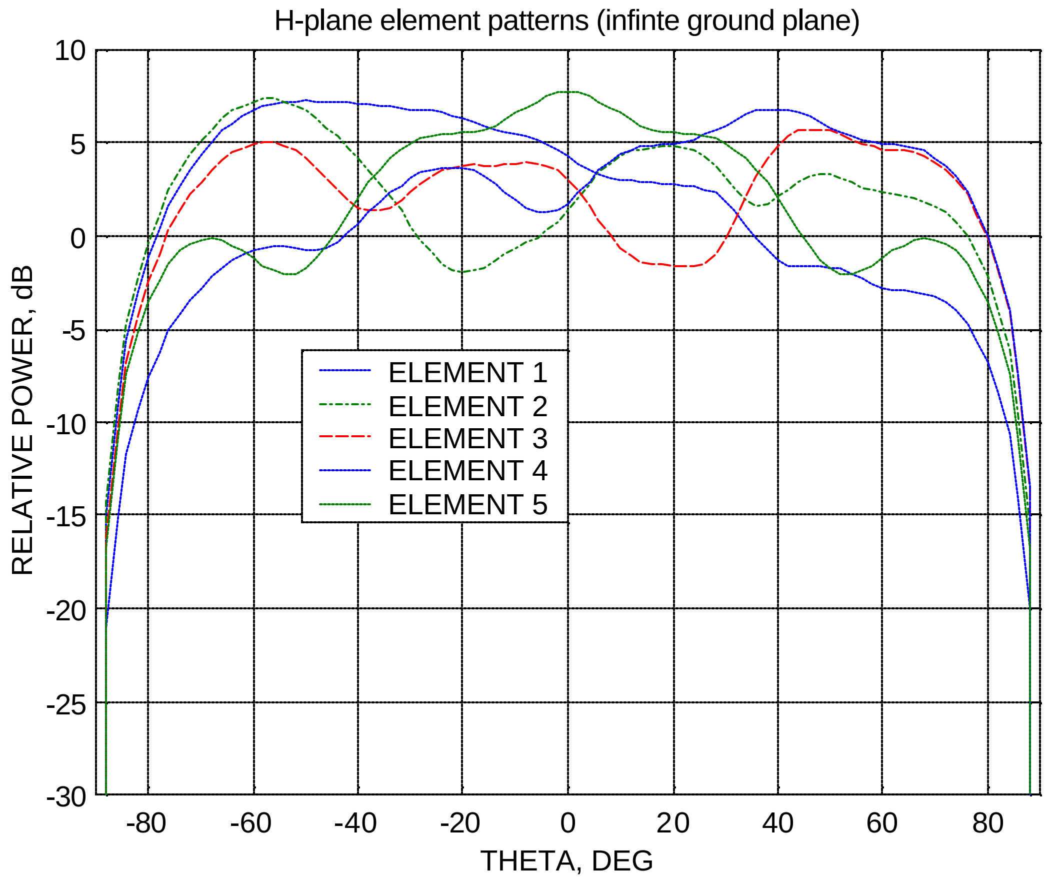


- A truncated ground plane affects the wide angle radiation in two ways:
  1. The image of an element near the edge of the ground plane is disrupted
  2. Diffraction occurs at the edges of the ground plane



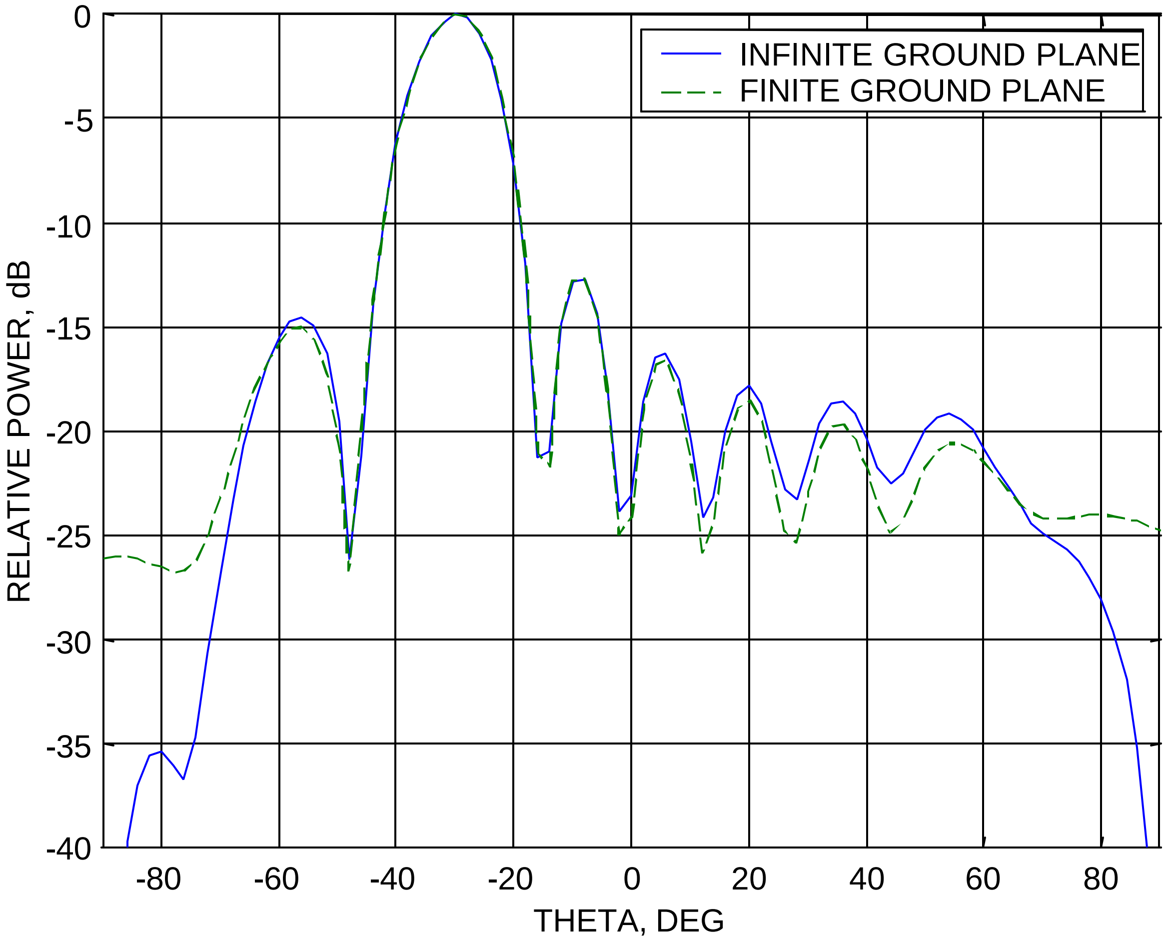
Finite Arrays and the "Edge Effect" (2)

Example of H-plane element patterns for a 10 element array (#1 is at an edge; #5 is near the center). The ground plane is infinite (i.e., there is no ground plane edge effect)

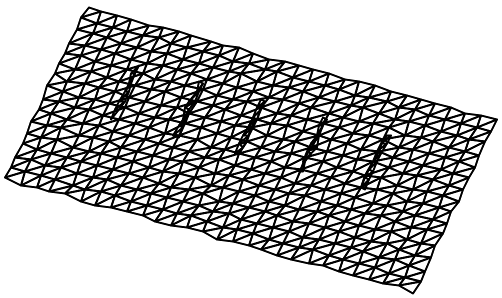


Finite Arrays and the “Edge Effect” (3)

These patterns illustrate the effect of a finite ground plane on the pattern of a finite array of 10 dipoles.

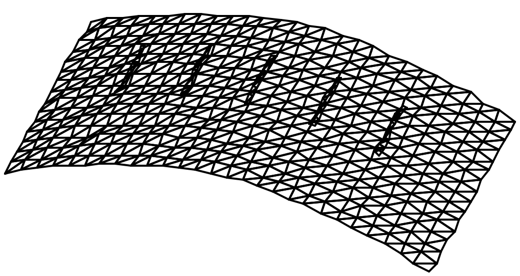


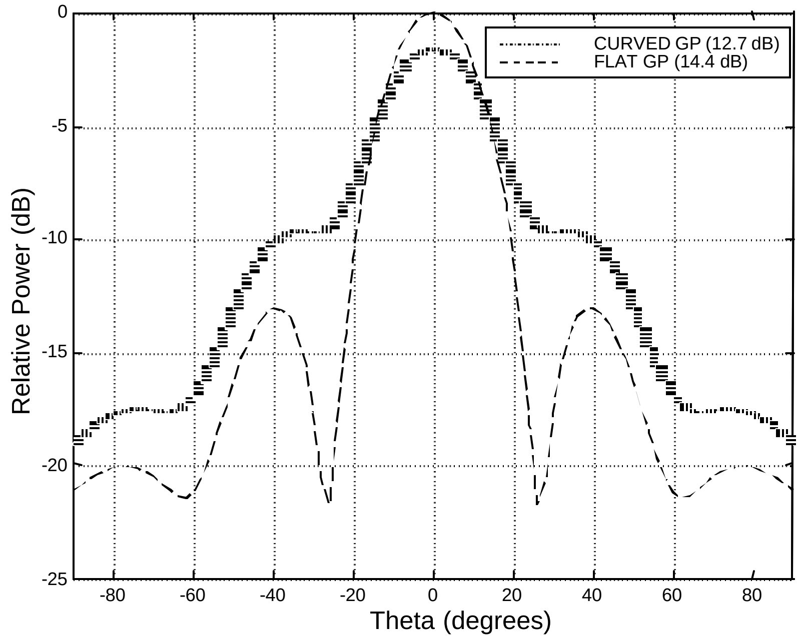
Curved vs. Flat Arrays



FLAT GROUND PLANE

CURVED GROUND PLANE

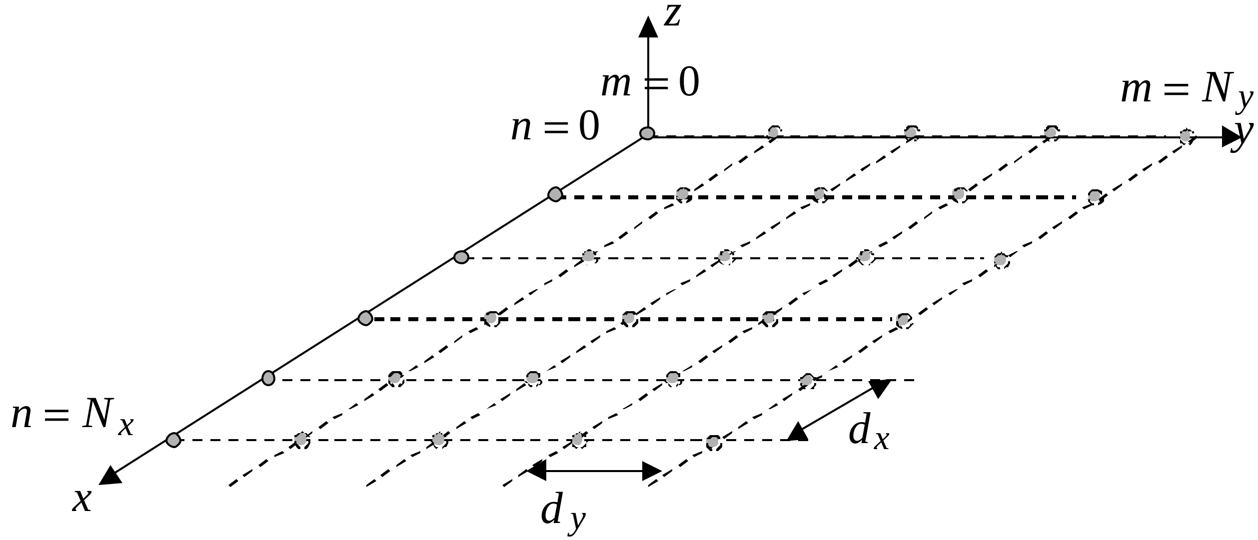




Two-dimensional Arrays (1)

A two-dimensional array can be formed by constructing a linear array whose elements are themselves linear arrays. In this case the element lattice (or geometrical arrangement) is rectangular. In general, a two-dimensional array can be constructed on any shaped surface. By far the most common situation is where the elements lie in a plane, i.e., a planar array.

Assume that an array is formed by linear arrays along the *x* axis, each having N_x elements spaced d_x. There are N_y linear arrays distributed uniformly along the *y* axis with spacing d_y.



Two-dimensional Arrays (2)

The array factor of the 2-d array is

$$|\mathrm{AF}| = \sum_{n=0}^{N_x} \sum_{m=0}^{N_y} A_{mn} e^{jnkd_x \sin\theta \cos\phi} e^{jmkd_y \sin\theta \sin\phi}$$

From the assumed feeding arrangement, $A_{mn} = A_m A_n$, which allows separation of the two sums

$$|\mathrm{AF}| = |\mathrm{AF}_x||\mathrm{AF}_y| = \sum_{n=0}^{N_x} A_n e^{jnkd_x \sin\theta \cos\phi} \sum_{m=0}^{N_y} A_m e^{jmkd_y \sin\theta \sin\phi}$$

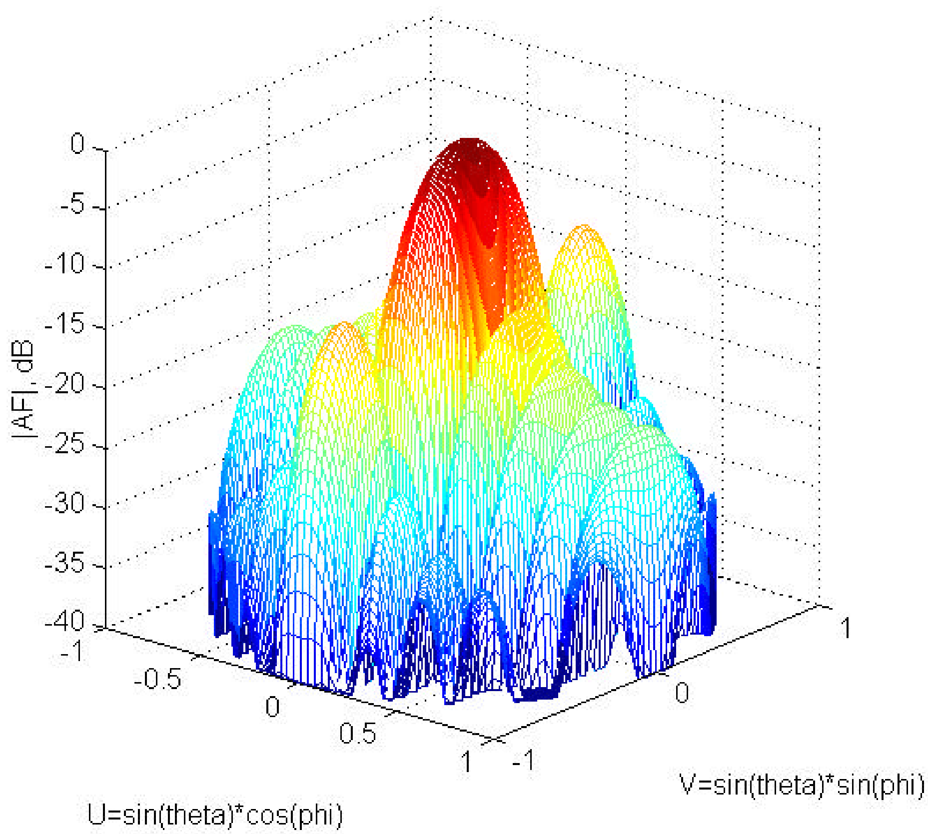
The sums are the same as in the linear array case

$$|\mathrm{AF}| = \left| \frac{\sin\left[\frac{N_x+1}{2} kd_x (\sin\theta \cos\phi - \sin\theta_s \cos\phi_s) \right]}{\sin[kd_x (\sin\theta \cos\phi - \sin\theta_s \cos\phi_s)/2]} \right| \left| \frac{\sin\left[\frac{N_y+1}{2} kd_y (\sin\theta \sin\phi - \sin\theta_s \sin\phi_s) \right]}{\sin[kd_y (\sin\theta \sin\phi - \sin\theta_s \sin\phi_s)/2]} \right|$$

The physical area of the array is approximately $A = (N_x+1)d_x (N_y+1)d_y$. The main beam direction is given by (θ_s, ϕ_s)

Two-dimensional Arrays (3)

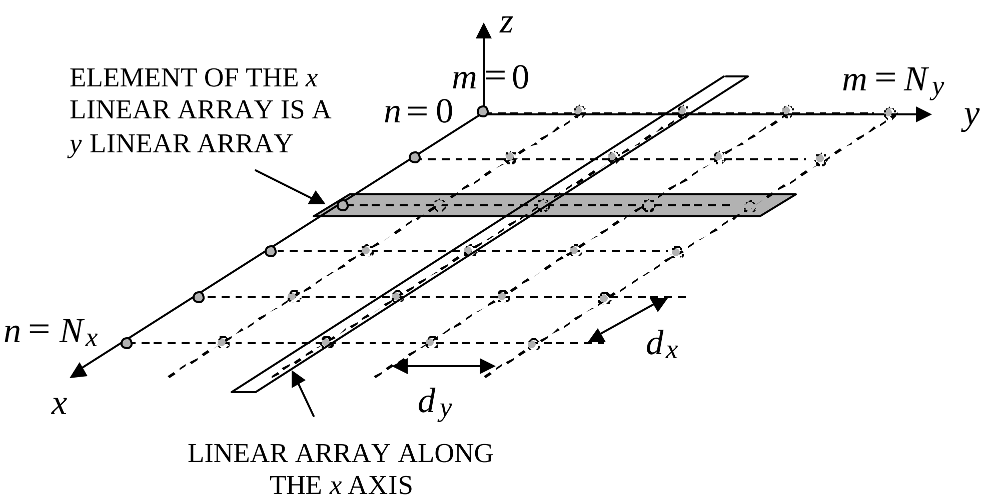
Mesh plot of a two dimensional array factor



$N_x = 10$
$N_y = 4$
$d_x = d_y = 0.5$
Uniform illumination
$(A_n = A_m = 1)$

Two-dimensional Arrays (4)

The directivity of a two-dimensional array can be obtained by considering it to be a linear array with "elements" that are themselves linear arrays.



Directivity of the x and y linear arrays for $l/2$ spacing:

$$D_{ox} = (N_x + 1)$$

$$D_{oy} = (N_y + 1)$$

Beamwidths of the linear arrays:

$$q_{Bx} \approx l / L_x = l / \big(d_x (N_x + 1)\big)$$

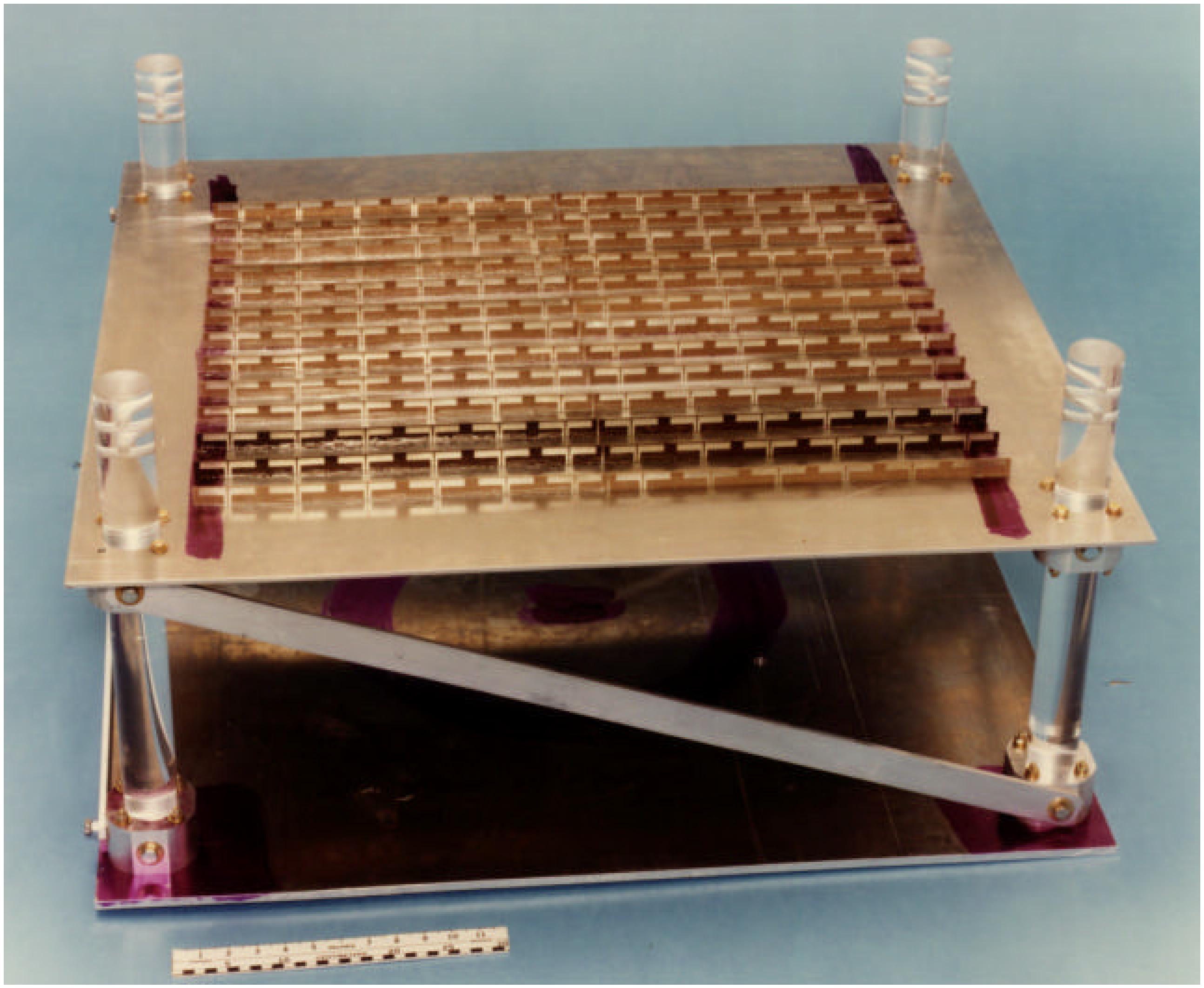
$$q_{By} \approx l / L_y = l / \big(d_y (N_y + 1)\big)$$

Using the approximate equation for directivity that was derived earlier

$$D_o = \frac{4p}{q_{Bx} q_{By}} = \frac{4p}{l^2} L_x L_y = \frac{4p A}{l^2}$$

This equation holds for any antenna that has an aperture with area A. It will be derived in a more rigorous manner later. The gain is $G = eD_o$, where e is the antenna efficiency.

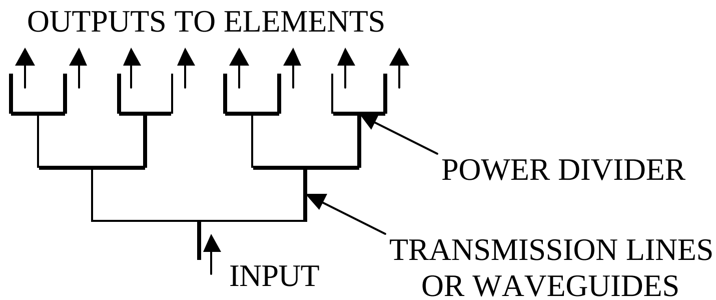
Two-Dimensional Array of Dipoles



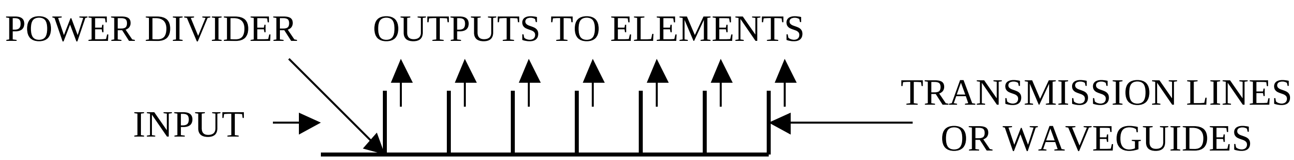
Microwave Beamforming Networks (1)

Beamforming networks are used to distribute a signal from the input to the individual element (or, on receive, to combine the signals from the elements and deliver it to the receiver). The networks fall into three broad categories:

1. Parallel feeds (or corporate feeds): they have a tree-like structure whereby the signal is split between branches.



2. Series feeds: the signal to the individual elements is tapped off of a main line.



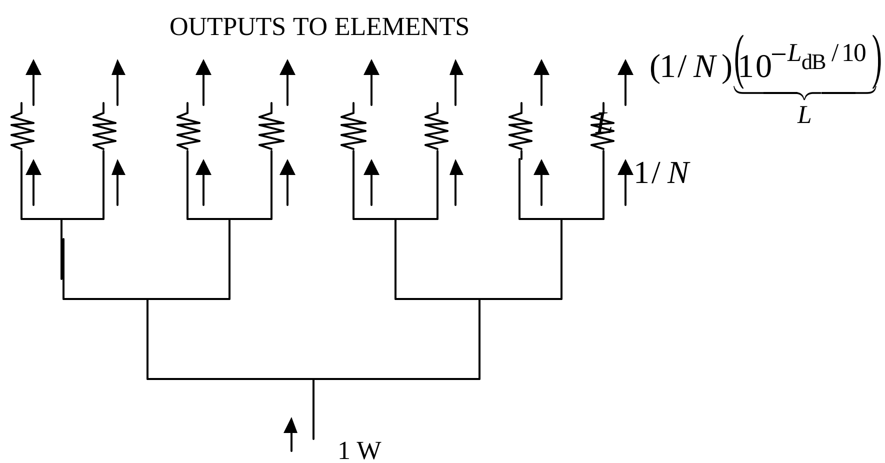
3. Space feeds: a region of space is used to combine or distribute signals. Space feeds generally incorporate reflectors or lenses

Microwave Beamforming Networks (2)

Example: A parallel feed splits the input power uniformly N ways. There is a cable loss of L_{dB} dB between the output of each branch and the element. What is the loss of this feed if the only loss is due to the cables?

If 1 W is applied to the input there is 1/N W at the input to each cable. The total power out is of the cables is the sum

$$P_{\mathrm{out}} = \sum_{n=1}^{N} (L/N) = L \sum_{n=1}^{N} (1/N) = L$$



The feed loss is

$$\frac{P_{\mathrm{out}}}{P_{\mathrm{in}}} = \frac{L}{1} = L = L_{\mathrm{dB}}$$

Note that the total loss is not NL_{dB} because the sources of the loss (resistors) are in parallel. This result does not hold for a series feed.

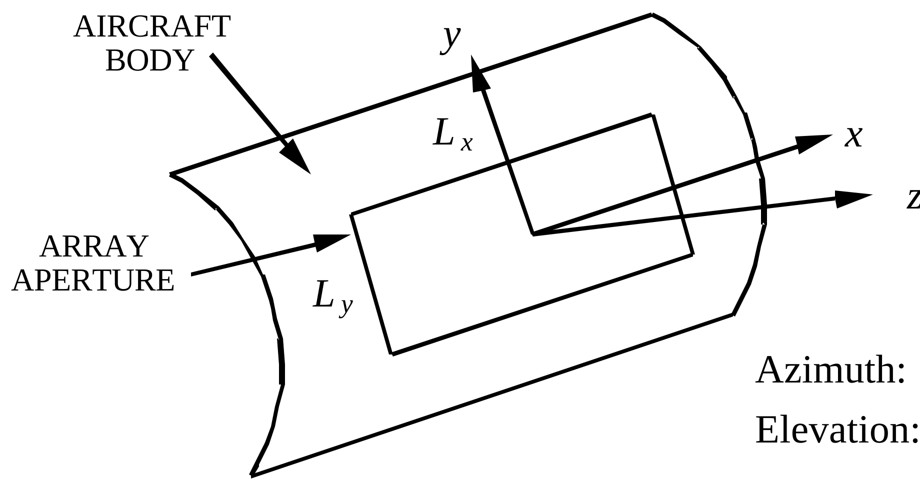
Corporate Fed Waveguide Array

Series Fed Waveguide Slot Arrays

Array Example (1)

Design an array to meet the following specifications:

1. Azimuth sidelobe level 30 dB
2. $\pm$ 45 degree scan in azimuth; no elevation scan; no grating lobes
3. Elevation HPBW of 5 degrees
4. Gain of at least 30 dB over the scan range



Azimuth: f=0°

Elevation: f=90°

Restrictions:

1. Elements are vertical ($\hat{y}$) dipoles over a ground plane
2. Feed network estimated to have 3 dB of loss
3. Dipole spacings are: $0.4l \le d_x \le 0.8l$ and $0.45l \le d_y \le 0.6l$

Array Example (2)

Restrictions (continued):

4. errors and imperfections will increase the SLL about 2 dB so start with a -32 dB sidelobe distribution

$$0.2 + 0.8\cos^2(\pi x'/2) \qquad (\epsilon_i = 0.81)$$

5. minimize the number of phase shifters used in the design

<u>Step 1</u>: start with the gain to find the required physical area of the aperture

$$G = D\epsilon = \frac{4\pi A}{\lambda^2}\epsilon\cos\theta \geq 30\,\mathrm{dB}$$

$\cos\theta$ is the projected area factor, which is a minimum at 45 deg. The efficiency includes tapering efficiency (0.81) and feed loss (0.5). Therefore

$$G = \frac{4\pi A}{\lambda^2}(0.707)(0.81)(0.5) = 10^3$$

or, $A/\lambda^2 = (L_x/\lambda)(L_y/\lambda) \approx (N_x+1)(d_x/\lambda)(N_y+1)(d_y/\lambda) = 278.1$

<u>Step 2</u>: uniform illumination in elevation; must have a HPBW of 5 degrees

$$\left|\frac{\sin\left((N_y+1)kd_y\sin\theta/2\right)}{(N_y+1)\sin\left(kd_y\sin\theta/2\right)}\right|_{\theta=2.5^\circ} = 0.707 \quad \Rightarrow \quad L_y = (N_y+1)d_y = 10\,\lambda$$

Array Example (3)

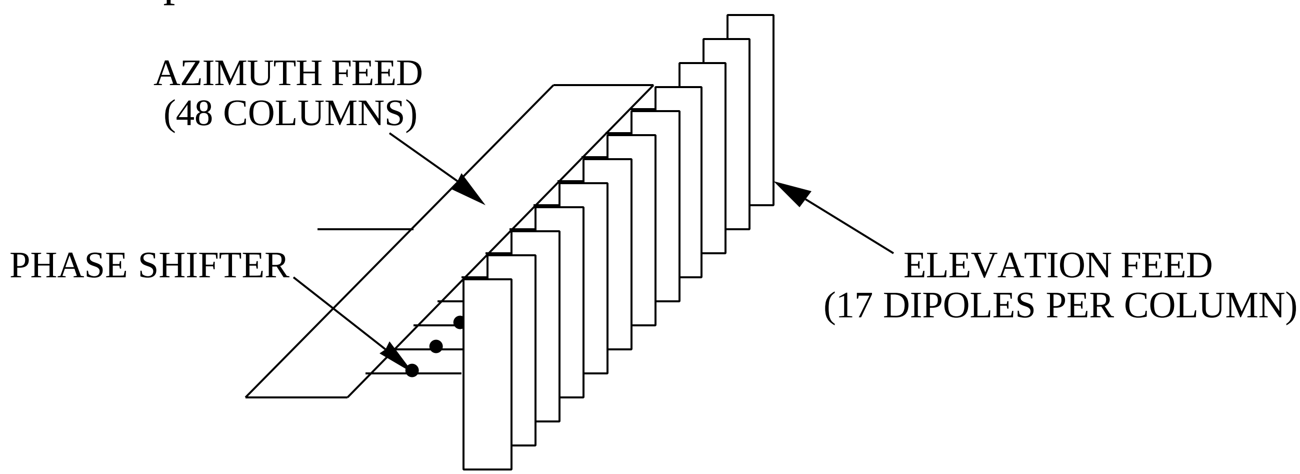
This leads to $L_x = A / L_y = 28 l$. To minimize the number of elements choose the largest allowable spacing

$$d_y = 0.6 l \quad \Rightarrow \quad N_y + 1 = L_y / d_y = 17$$

<u>Step 3</u>: azimuth spacing must avoid grating lobes which occur when

$$\sin q_n - \sin q_s = n l \,/\, d_x \qquad (q_n \le -90^\circ,\, n = -1,\, q_s = 45^\circ)$$
$$-1 - 0.707 = -l \,/\, d_x \quad \Rightarrow \quad d_x \le 0.585 l$$

Again, to minimize the number of elements, use the maximum allowable spacing ($0.585 l$) which gives $N_x + 1 = L_x / d_x = 48$. Because the beam only scans in azimuth, one phase shifter per column is sufficient.



Array Example (4)

Step 4: Find the azimuth beamwidth at scan angles of 0 and 45 degrees. Letting $q_H = q_B / 2$

$$\left| \frac{\sin\left((N_x + 1) k d_x (\pm \sin q_H - \sin q_s)/2\right)}{(N_x + 1)\sin\left(k d_x (\pm \sin q_H - \sin q_s)/2\right)} \right| = 0.707$$

Solve this numerically for $q_s = 0$ and 45 degrees. Note that the beam is not symmetrical when it is scanned to 45 degrees. Therefore the half power angles are different on the left and right sides of the maximum

$$q_s = 0: \quad q_{B^+} = -q_{B^-} = 0.91 \quad \Rightarrow \quad \text{HPBW} = q_{B^+} - q_{B^-} = 2(0.91) = 1.82^\circ$$

$$q_s = 45: \quad q_{B^+} = 46.3, \quad q_{B^-} = 43.75 \quad \Rightarrow \quad \text{HPB}W = 46.3 - 43.75 = 2.55^\circ$$

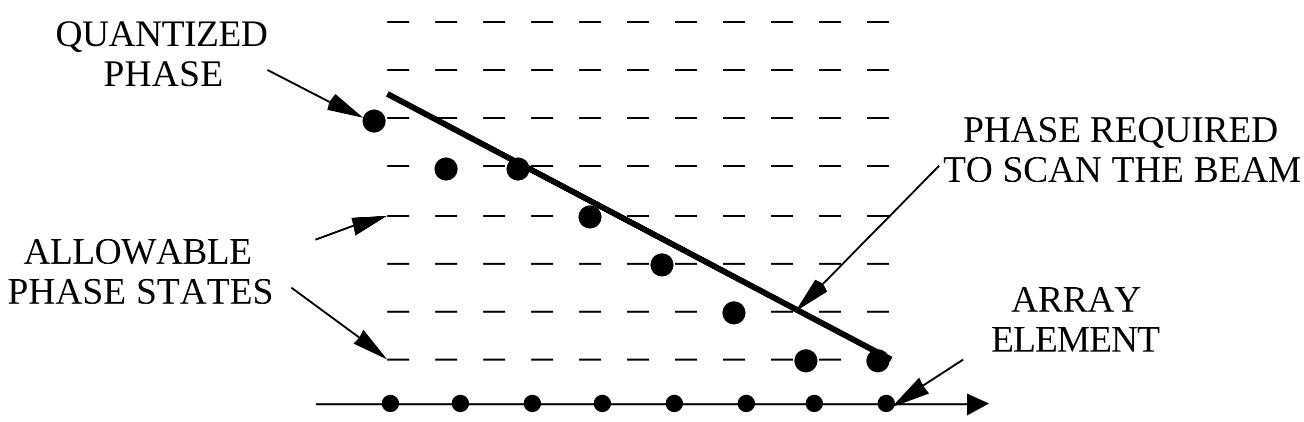
We have not included the element factor, which will affect the HPBW at 45 degrees.

Digital Phase Shifters

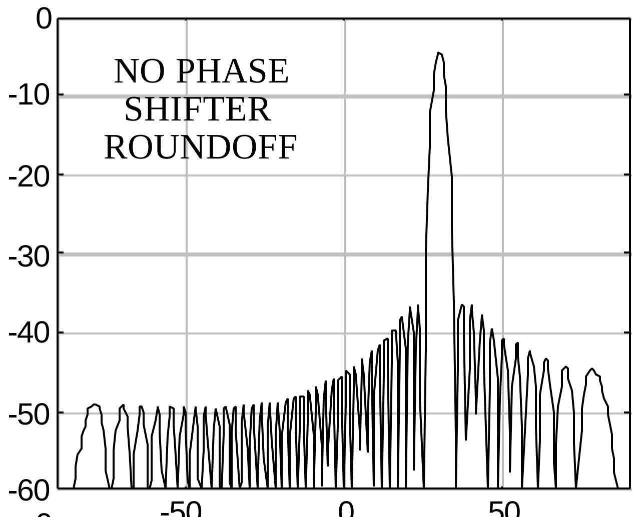
Phase shifters introduce a precise phase shift to a wave passing through it. They are used to "tilt" the phase across the array aperture for beam scanning. Diode phase shifters are capable of providing only discrete phase intervals. A n bit phase shifter has 2^n phase states. The quantization levels are separated by $\Delta = 360^\circ / 2^n$. The fact that the exact phase cannot always be obtained results in:

1. gain loss
2. increase in sidelobe level
3. beam pointing error

Example of phase truncation:

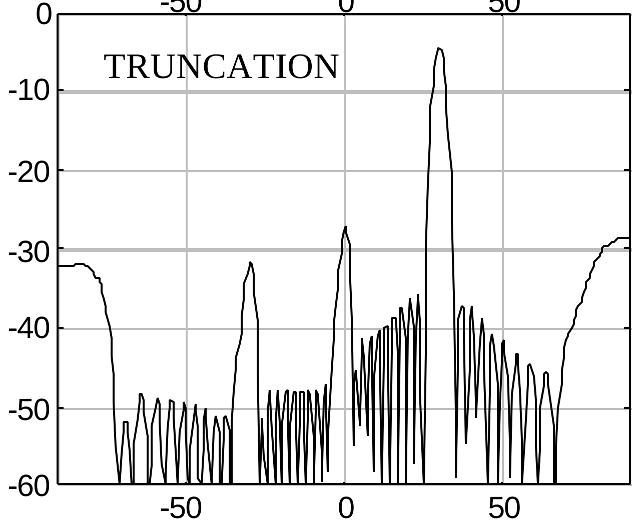


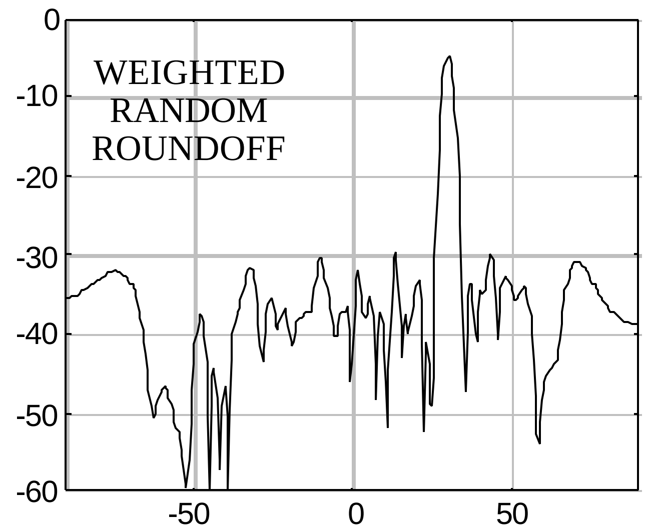
Effect of Phase Shifter Roundoff Errors



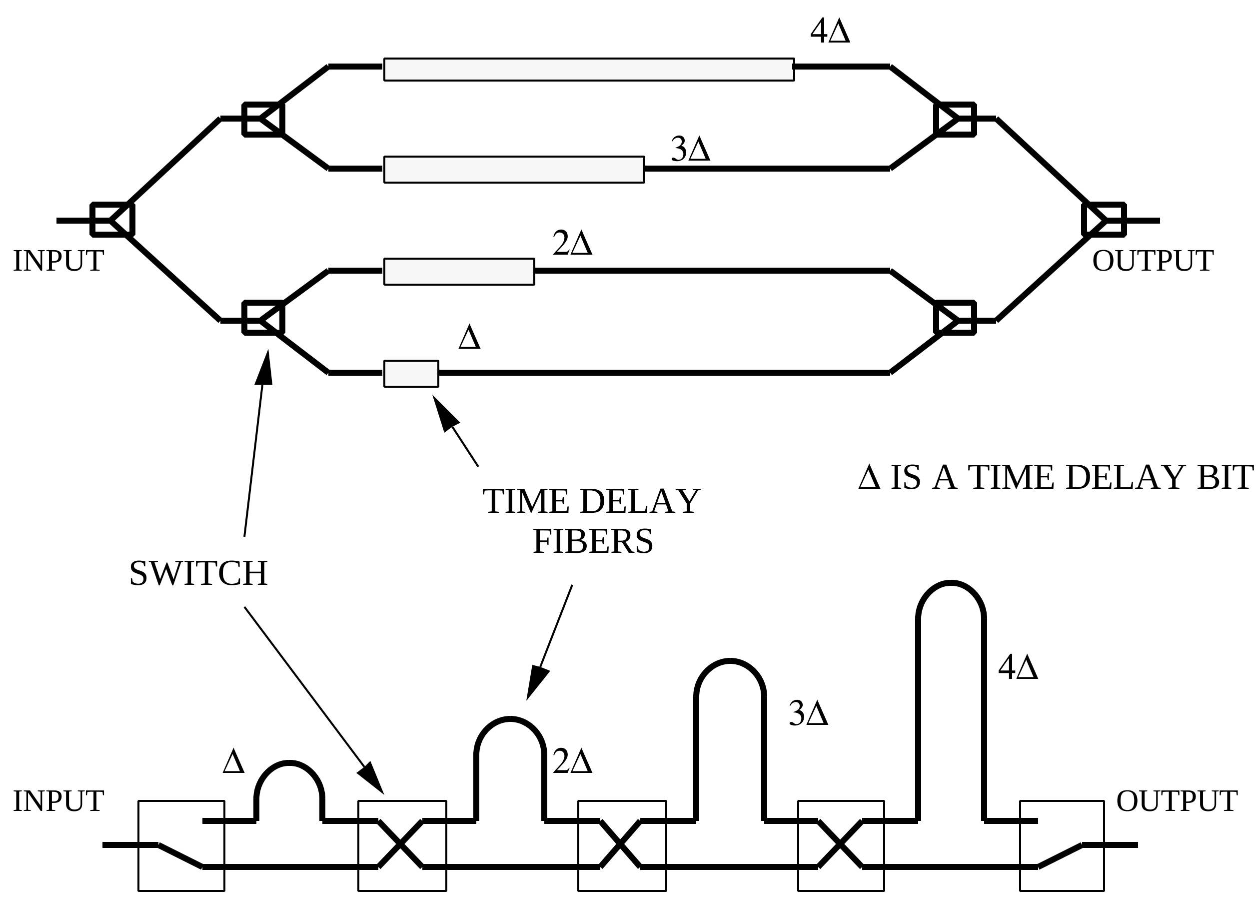
Truncation causes beam pointing errors. Random roundoff methods destroy the periodicity of the quantization errors. The resultant rms error is smaller than the maximum error using truncation.

Linear array, 60 elements, $d = 0.4\lambda$
4 bit phase shifters



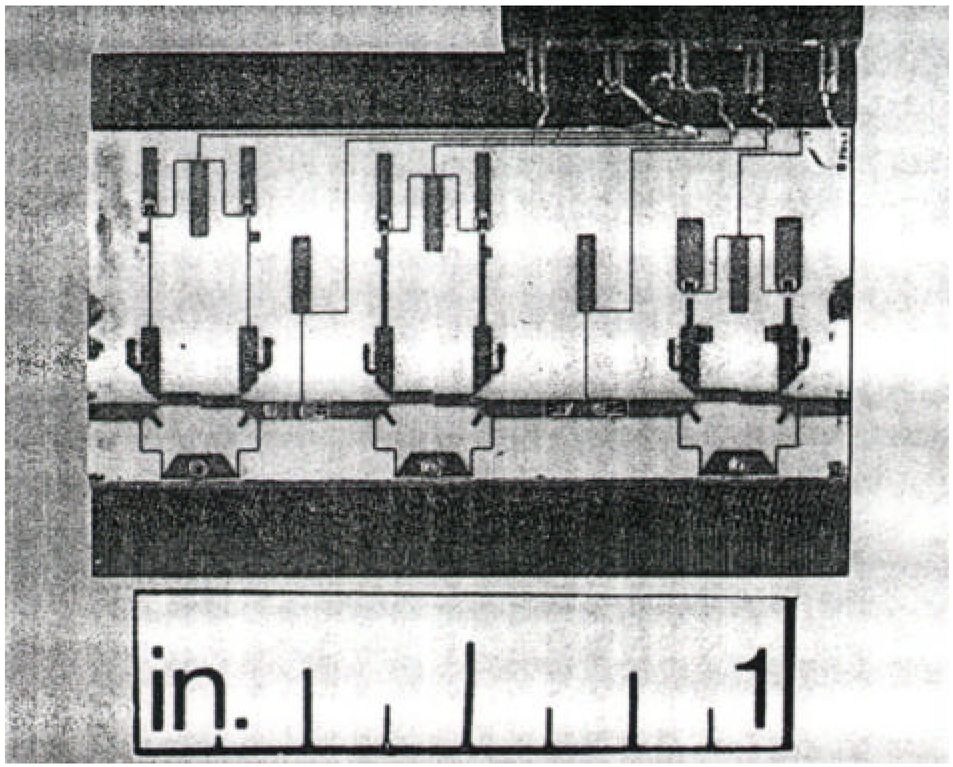


Examples of Time Delay Networks



Digital Phase Shifter

- X-band 5-bit PIN diode phase shifter



From Hughes Aircraft Co.(Raytheon)

Discrete Fourier Transform (1)

The discrete Fourier transform (DFT) is a sampled version of the continuous Fourier transform

$$F(u_m) = \sum_{n=0}^{N-1} f(x_n) e^{-j\left(\frac{2p}{N}\right)mn} \qquad (m = 0,1,\ldots,N-1)$$

The corresponding inverse discrete Fourier transform (IDFT) is

$$f(x_n) = \frac{1}{N} \sum_{n=0}^{N-1} F(u_m) e^{j\left(\frac{2p}{N}\right)mn} \qquad (n = 0,1,\ldots,N-1)$$

$F(m)$ and $f(n)$ are sometimes used to denote sampled data. The <u>fast Fourier transform</u> (FFT) is an efficient means of calculating the DFT because it dramatically reduces the number of required arithmetic operations. Clearly the DFT and IDFT calculations are similar, so the FFT can be used to compute the IDFT as well.

In general, the array factor for elements along the *x* axis is

$$\mathrm{AF}(q,f) = \sum_{n=0}^{N-1} A(x_n)\, e^{j x_n k \sin q \cos f}$$

Discrete Fourier Transform (2)

For simplicity, let the observation point lie in the *x*-*z* plane ($f = 0^\circ$). Evaluating the array factor at a set of observation angles q_m, $m = 0,1,\ldots, N$-1

$$\mathrm{AF}(q_m) = \sum_{n=0}^{N-1} A_n \, e^{j x_n k \sin q_m} = \sum_{n=0}^{N-1} A_n \, e^{j x_n k u_m}$$

where $u_m = \sin q_m$ is the *x* direction cosine. If Δu is the spacing between pattern values in direction cosine space, then the array factor is

$$\mathrm{AF}(u_m) = \sum_{n=0}^{N-1} A_n \, e^{j(nd)(km\Delta u)} = \sum_{n=0}^{N-1} A_n \, e^{jmn(kd\Delta u)}$$

The summation conforms to the definition of the IDFT

$$\mathrm{AF}(m) = \sum_{n=0}^{N-1} A(n) e^{-j\left(\frac{2p}{N}\right)mn} \qquad (m = 0,1,\ldots, N-1)$$

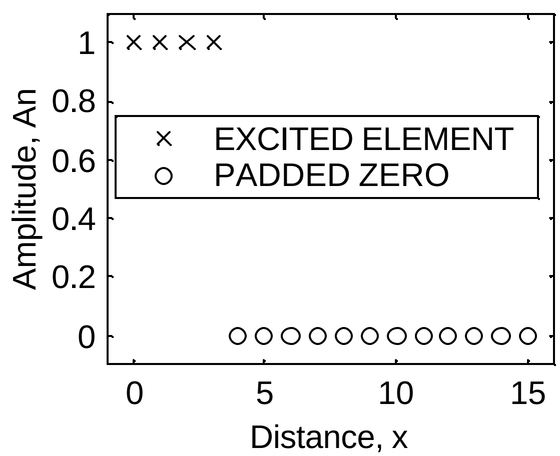
where $kd\Delta u = 2p / N$, or $\Delta u = \dfrac{2p}{Nkd} = \dfrac{l}{Nd}$. The FFT algorithm will return $N/2$ negative values of *u*, $N/2-1$ positive values of *u*, and $u = 0$ (for a total of *N*).

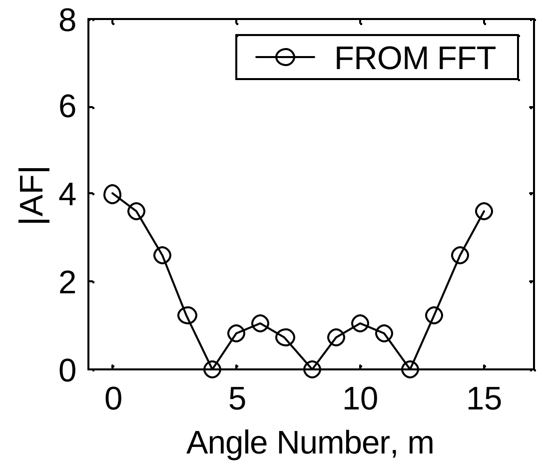
Discrete Fourier Transform (3)

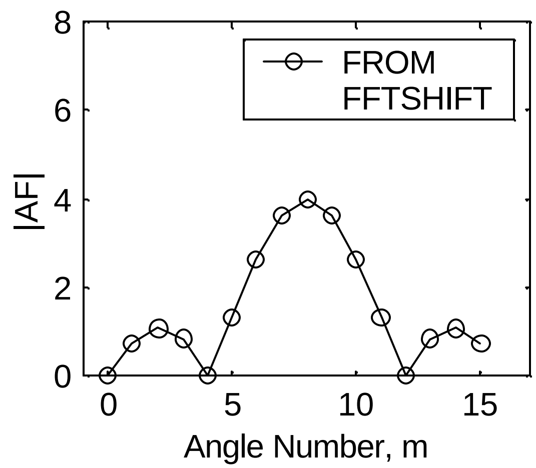
The FFT computation has the following properties:

1. Most software is based on the original <u>radix 2 algorithm</u> which requires the number of inputs (aperture distribution samples) to satisfy $N = 2^M$ where *M* is an integer. In Matlab when *M* is not an integer a less efficient calculation is used.
2. The number of outputs (pattern angles) is equal to the number of inputs.
3. A characteristic of the algorithm is a wrap-around of the output data. In Matlab the output data is arranged in the proper order by calling FFTSHIFT on the array returned by FFT.
4. If more pattern angles than aperture samples are desired (usually the case) then the input can be appended with zeros.
5. The FFT returns *N* samples of half of a period of the array factor. In other words, if the aperture samples are taken at $0.5l$ intervals (i.e., the array elements are spaced $0.5l$), then the samples will correspond to the range $-90^\circ \le q \le 90^\circ$ or equivalently $-1 \le kd \le 1$.
6. If the element spacing is less than $0.5l$ then some of the output data must be discarded; if the element spacing is greater than $0.5l$ then more periods of the AF must be generated. This is accomplished by inserting one or more zeros between the aperture distribution samples.

Discrete Fourier Transform (4)



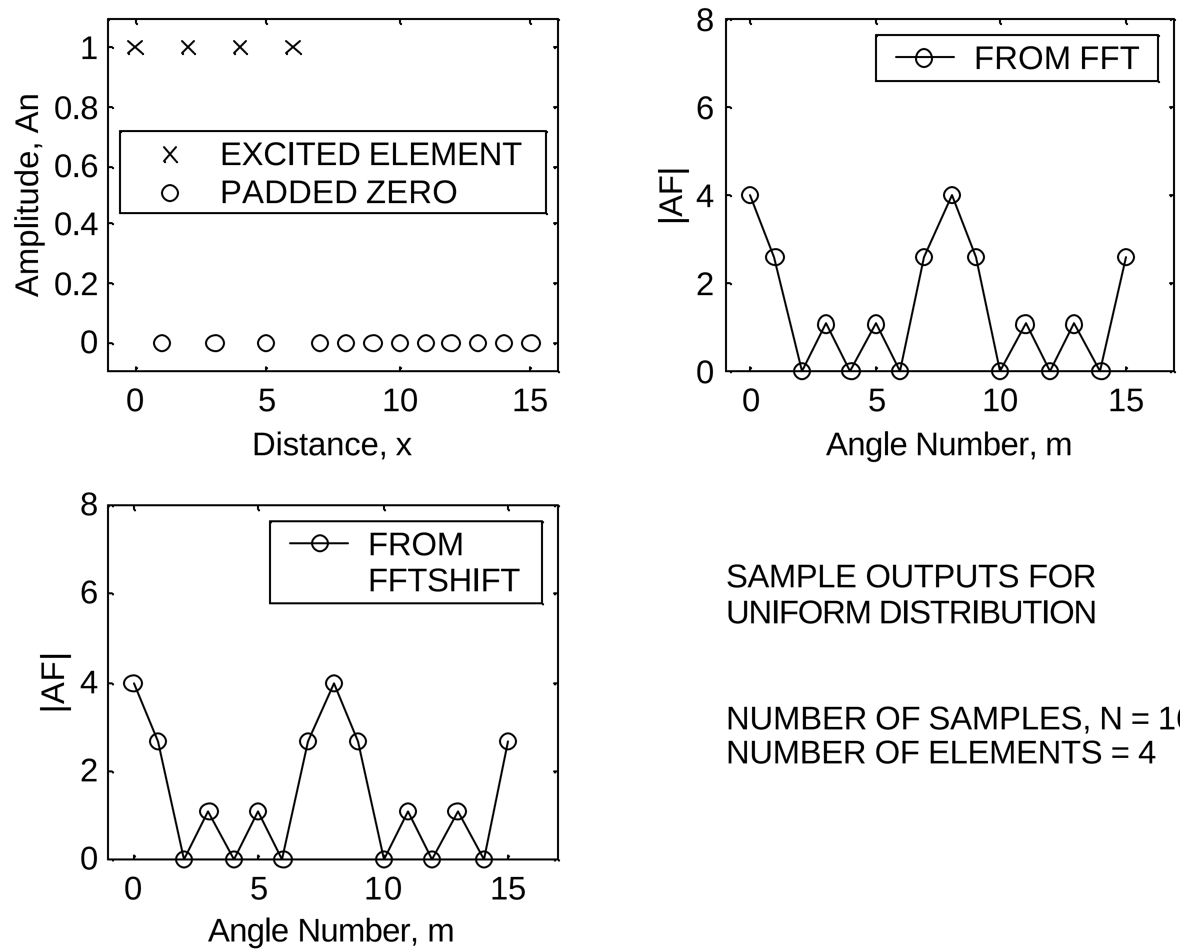




SAMPLE OUTPUTS FOR UNIFORM DISTRIBUTION

NUMBER OF SAMPLES, N = 16
NUMBER OF ELEMENTS = 4

Discrete Fourier Transform (5)



SAMPLE OUTPUTS FOR
UNIFORM DISTRIBUTION

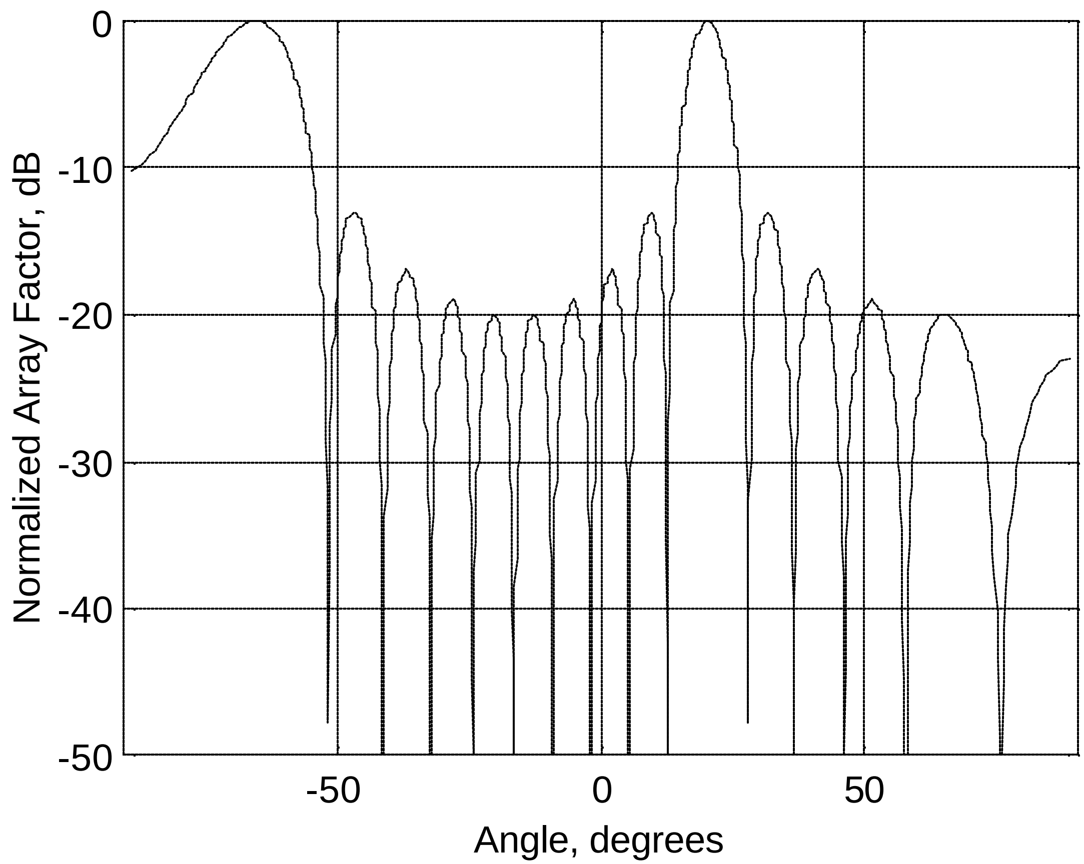
NUMBER OF SAMPLES, N = 16
NUMBER OF ELEMENTS = 4

Matlab Program to Compute the Array Factor

```
% program to compute the array factor for a
% linear array of elements using the MATLAB
% FFT routine
clear,clf
rad=pi/180;
M=11;
N=2^M;
d=0.5;        % spacing in wavelengths
Nel=10;       % number of elements
thetas=20;    % scan angle
% phase for scanning the beam
psis=2*pi*d*sin(thetas*rad);
% number of periods required from the
spacing
  npack=0;
   while (npack/2/d) < 1
    npack=npack+1;
   end
% assign excitation values and pad the
% distribution with zeros
   A=zeros(N,1);
 for k=1:Nel
  A(npack*k)=1*exp(j*(k-1)*psis);
 end
 z=fft(A,N);
 y=fftshift(z);
 ydb=20*log10(abs(y)/Nel+1e-2);
 nmin=N/2; nmax=N/2;
% throw away output values that are not in
% the visible region for this spacing
 for k=1:N
   arg=(k-N/2)*npack/N/d;
    if arg < 1
     nmax=max(nmax,k);
    end
    if arg > -1
     nmin=min(nmin,k);
    end
    if abs(arg) < 1
     x(k)=atan(arg/sqrt(1-arg^2))/rad;
    end
  end
  for k=nmin:nmax
   AFdb(k-nmin+1)=ydb(k);
   theta(k-nmin+1)=x(k);
 end
 plot(theta,AFdb),grid
 axis([-90,90,-50,0])
 xlabel('Angle, degrees')
 ylabel('Normalized Array Factor, dB')
```

Sample Output of the Matlab Program

Sample calculation for 20 elements, $d = 0.8\lambda$, $N = 2048$ (FFT samples), $\theta_s = 20^\circ$. A grating lobe occurs in the visible region for this spacing.

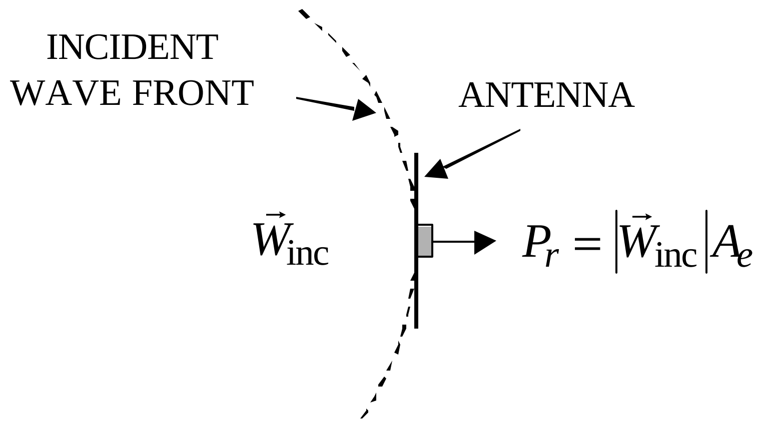


Receiving Antennas (1)

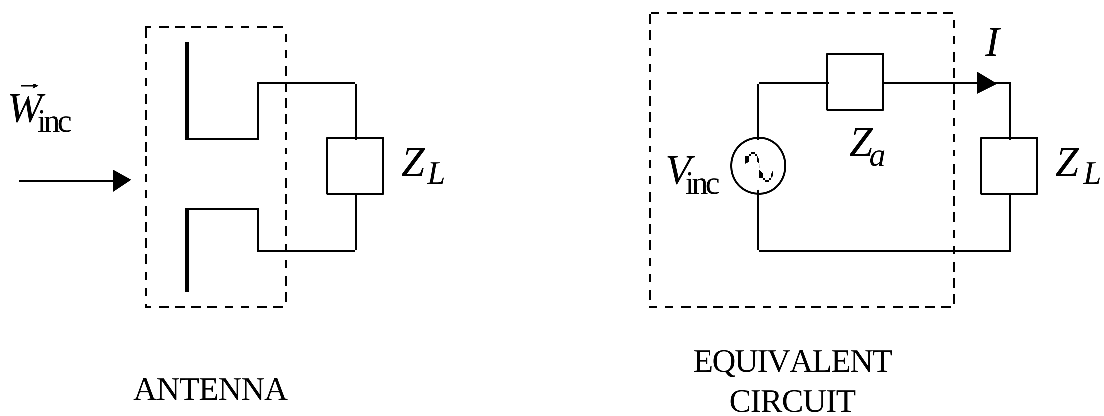
When an antenna is receiving, it is convenient to define an <u>effective area</u> (or <u>effective aperture</u>) A_e. The power delivered to a load at the antenna terminals is

$$P_r = \left|\vec{W}_{\text{inc}}\right| A_e$$

where $\vec{W}_{\text{inc}}$ is the incident power density.



An equivalent circuit for the antenna is shown below. The current is $I = V_{\text{inc}} / (Z_a + Z_L)$.

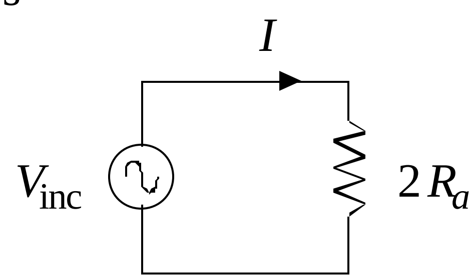


Receiving Antennas (2)

Let the load be conjugate matched to the antenna impedance (which is the condition for maximum power transfer) and assume there are no losses ($R_\ell = 0$)

$$Z_L = Z_a^* \quad (R_L = R_a \text{ and } X_L = -X_a).$$

The equivalent circuit becomes



The power delivered to the receiver can be found in terms of the effective area

$$P_r = \frac{1}{2} I^2 R_a = \frac{1}{2} \frac{V_{\text{inc}}^2}{4R_a} \equiv \left|\vec{W}_{\text{inc}}\right| A_e$$

For a Hertzian dipole $R_a = \dfrac{h(k\ell)^2}{6p}$, $E_{\text{inc}} = V_{\text{inc}} / \ell$, and $\left|\vec{W}_{\text{inc}}\right| = \dfrac{1}{2} \dfrac{E_{\text{inc}}^2}{h}$. Now solve for A_e.

Receiving Antennas (3)

$$A_e = \frac{1}{2}\frac{V_{\text{inc}}^2}{(4R_a)\left|\vec{W}_{\text{inc}}\right|} = \frac{1}{2}\frac{V_{\text{inc}}^2}{(4R_a)\left[\frac{1}{2}\frac{E_{\text{inc}}^2}{h}\right]} = \frac{3}{2}\frac{p\, l^2}{(2p)^2} = 0.119\, l^2$$

For a Hertzian dipole the directivity is 3/2, and therefore the effective area can be written as

$$A_{e_m} = \frac{3}{2}\left(\frac{l^2}{4p}\right) = D\left(\frac{l^2}{4p}\right) \quad \Rightarrow \quad D = \frac{4p\, A_{e_m}}{l^2}$$

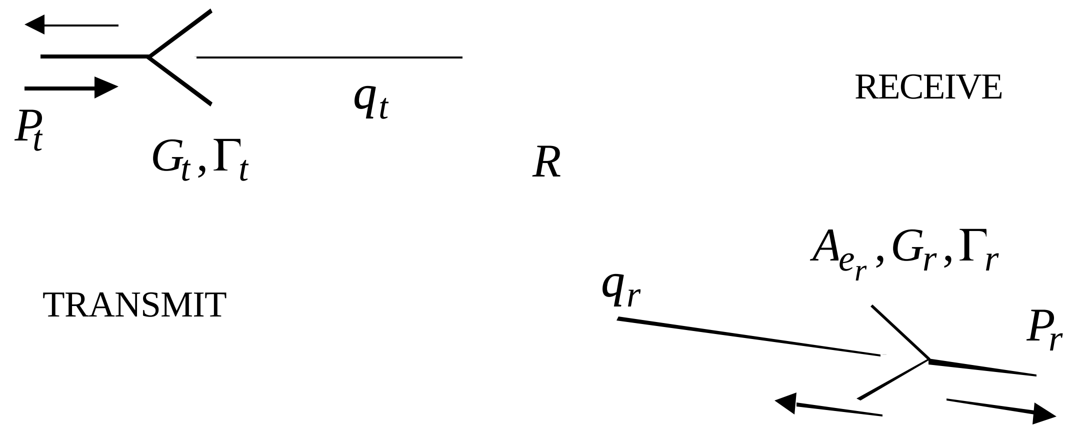
The subscript *m* denotes that it is the maximum effective area because the losses are not included. If losses are included then the gain is substituted for directivity

$$A_e = G\left(\frac{l^2}{4p}\right) \quad \Rightarrow \quad G = \frac{4p\, A_e}{l^2}$$

The formula holds for any type of antenna that has a well-defined aperture, or surface area through which all of the radiated power flows. From the formula one can deduce that the effective area is related to the physical area A by $A_{e_m} = eA$.

Friis Transmission Equation (1)

Consider two antennas that form a communication or data link. The range between the antennas is *R*. (The pattern can depend on both q and f, but only q is indicated.)



The power density at the receive antenna is

$$\left|\vec{W}_{\text{inc}}\right| = \frac{\overbrace{P_t\left(1-|\Gamma_t|^2\right)}^{\text{POWER INTO ANTENNA}}}{4pR^2} G_t(q_t)$$

and the received power is $P_r = \left|\vec{W}_{\text{inc}}\right| A_{er}(q_r)\left(1-|\Gamma_r|^2\right)p$ (p is the polarization loss factor, PLF).

Friis Transmission Equation (2)

But $A_{e_r} = G_r(q_r)\,l^2/(4p)$,

$$P_r = \frac{P_t G_t(q_t) G_r(q_r)\,l^2}{(4pR)^2}\left(1-|\Gamma_t|^2\right)\left(1-|\Gamma_r|^2\right)pL$$

L is a general loss factor ($0 \le L \le 1$). This is known as the <u>Friis transmission equation</u> (sometimes called the <u>link equation</u>).

Example: (Satcom system) Parameters at the ground station (uplink):

$G_t = 54$ dB $= 251188.6$, $L = 2$ dB $= 0.6310$, $P_t = 1250$ W
R=23,074 miles = 37,132 km, f =14 GHz

At the satellite (downlink)

$G_r = 36$ dB $= 3981$, $P_t = 200$ W, f =12 GHz

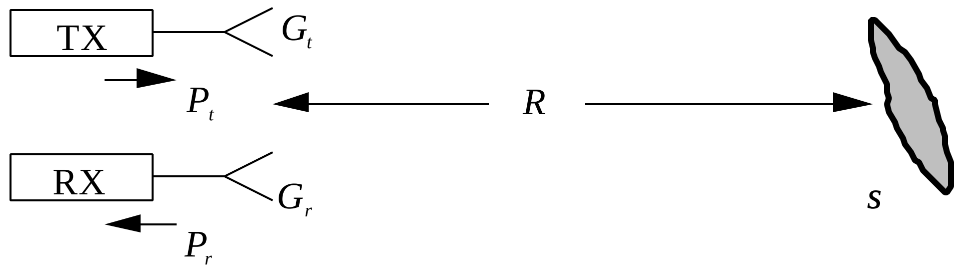
If we assume polarization matched antennas and no reflection at the antenna inputs,

$$P_r = \frac{(1250)(251188.6)(3981)(0.0214)^2}{(4p)^2\left(37132\times10^3\right)^2}(0.6310)$$

$$= 1.66\times10^{-9}\ \text{W} = -87.8\,\text{dBw} = -57.8\,\text{dBm}$$

Radar Range Equation (1)

"Quasi-monostatic" geometry:



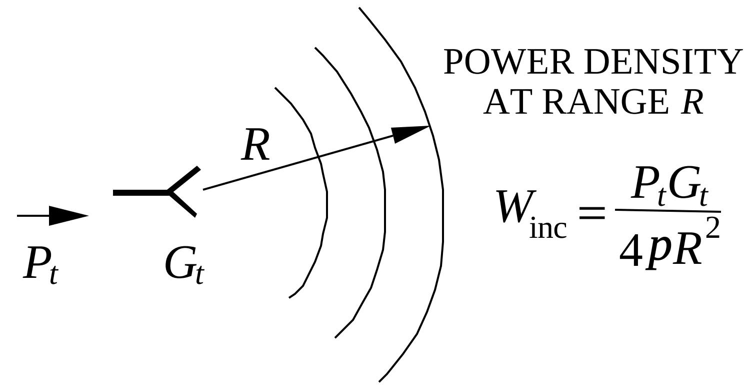
s = radar cross section (RCS) in square meters
P_t = transmitter power, watts
P_r = received power, watts
G_t = transmit antenna gain in the direction of the target (assumed to be the maximum)
G_r = transmit antenna gain in the direction of the target (assumed to be the maximum)
$P_t G_t$ = effective radiated power (ERP)

From antenna theory: $G_r = \dfrac{4pA_{er}}{l^2}$

- $A_{er} = Ae$ = effective area of the receive antenna
- A = physical aperture area of the antenna
- l = wavelength (c / f)
- e = antenna efficiency

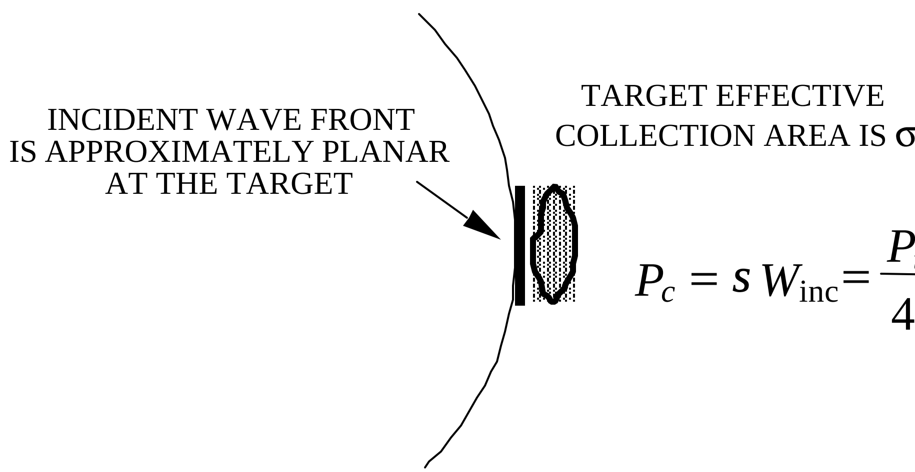
Radar Range Equation (2)

Power density incident on the target, $\vec{W}_{\text{inc}}$



$$W_{\text{inc}} = \frac{P_t G_t}{4\pi R^2} \quad (\text{W}/\text{m}^2)$$

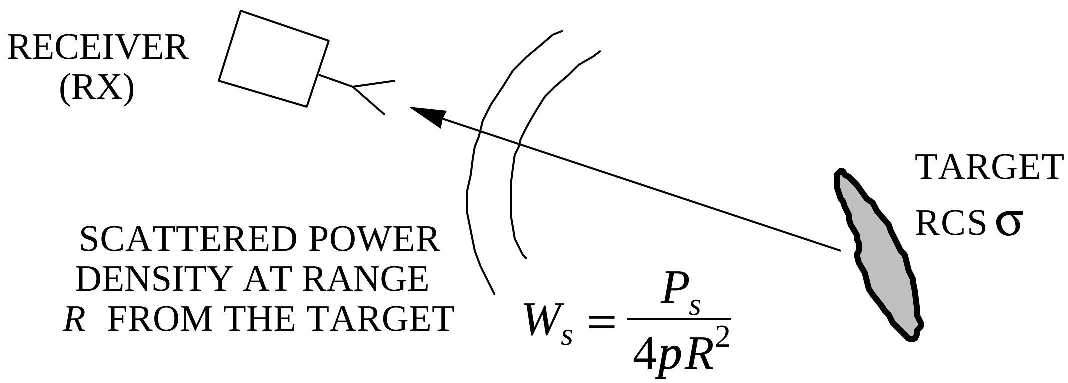
Power collected by the radar target and scattered back towards the radar



$$P_c = \sigma W_{\text{inc}} = \frac{P_t G_t \sigma}{4\pi R^2}$$

Radar Range Equation (3)

The RCS gives the fraction of incident power that is scattered back toward the radar. Therefore, $P_s = P_c$ and the scattered power density at the radar, $\vec{W}_s$, is obtained by dividing by $4\pi R^2$.



The target scattered power collected by the receive antenna is $W_s A_{er}$. Thus the maximum target scattered power that is available to the radar is

$$P_r = \frac{P_t G_t \sigma A_{er}}{(4\pi R^2)^2} = \frac{P_t G_t G_r \sigma \lambda^2}{(4\pi)^3 R^4}$$

This is the classic form of the radar range equation (RRE).

Radar Range Equation (4)

Including the reflections at the antenna terminals

$$P_r = \frac{P_t G_t \sigma A_{er}}{(4\pi R^2)^2}\left(1-|\Gamma_t|^2\right)\left(1-|\Gamma_r|^2\right)L = \frac{P_t G_t G_r \sigma \lambda^2}{(4\pi)^3 R^4}\left(1-|\Gamma_t|^2\right)\left(1-|\Gamma_r|^2\right)L$$

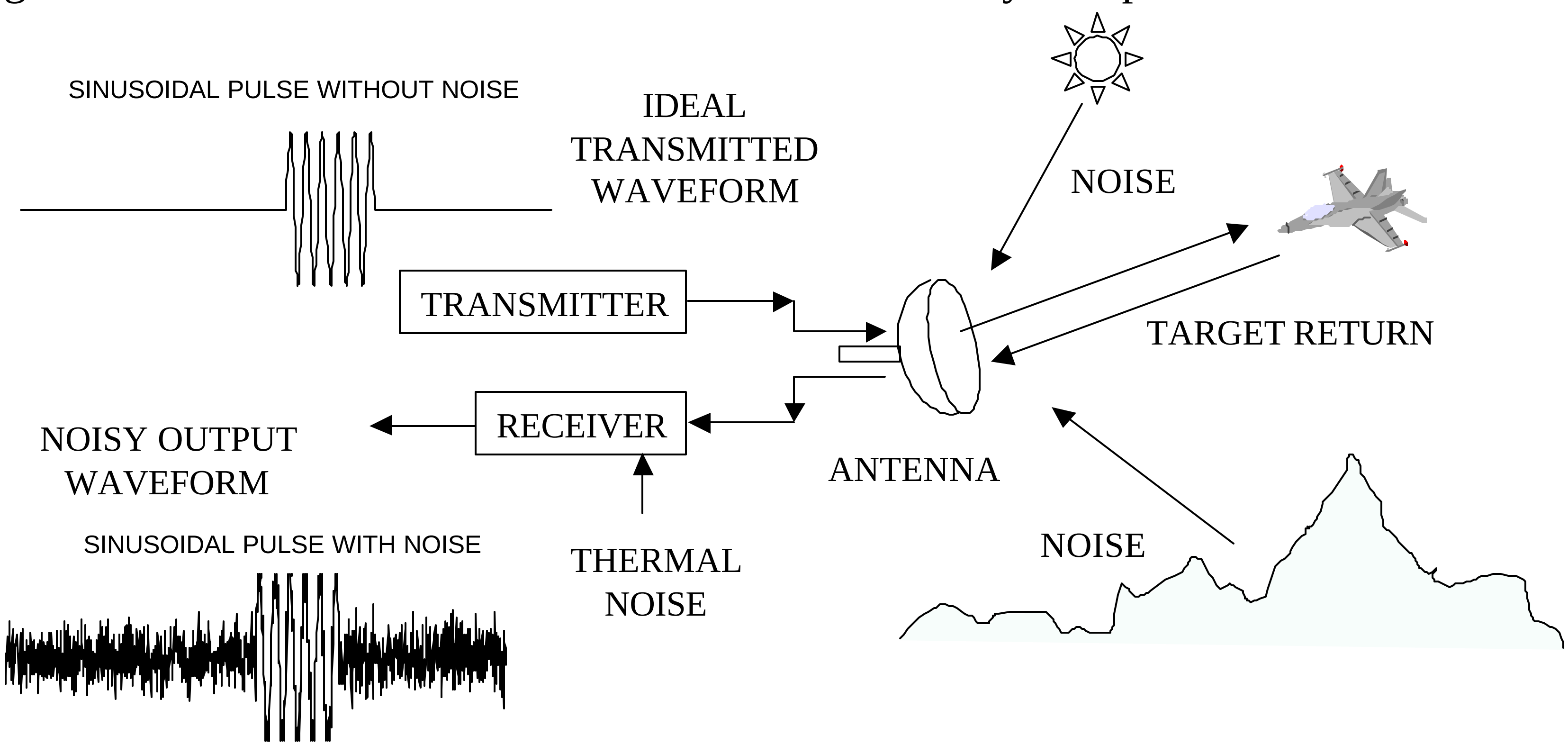
For monostatic systems a single antenna is generally used to transmit and receive so $G_t = G_r \equiv G$ and $\Gamma_r = \Gamma_t$. The above form of the RRE is too crude to use as a design tool. Factors have been neglected that have a significant impact on radar performance:

- noise,
- system losses,
- propagation behavior,
- clutter,
- waveform limitations, etc.

However, this form of the RRE does give some insight into the tradeoffs involved in radar design. The dominant feature of the RRE is the $1/R^4$ factor. Even for targets with relatively large RCS, high transmit powers must be used to overcome the $1/R^4$ when the range becomes large.

Noise in Systems (1)

One way that noise enters communication and radar systems is from background radiation of the environment. (This refers to emission by the background as opposed to scattering of the system's signal by the background, which is clutter.) Noise is also generated by the components in the radar's receive channel. Under most conditions it is the internally generated thermal noise that dominates and limits the system performance.



Noise in Systems (2)

A high noise level will hide a weak signal and possibly cause a loss in communications or, in the case of radar, prevent detection of a target with a low radar cross section.

- Thermal noise is generated by charged particles as they conduct. High temperatures result in greater thermal noise because of increased particle agitation.
- Noise is a random process and therefore probability and statistics must be invoked to access the impact on system performance.
- Thermal noise exists at all frequencies. We will consider the noise voltage to be constant with frequency (so called <u>white noise</u>) and its statistics (average and variance) independent of time (<u>stationary</u>).

If the noise voltage generated in a resistor at temperature T Kelvin (K) is measured, it is found to obey Plank's blackbody radiation law

$$V_n = \sqrt{\frac{4hfBR}{e^{hfk\mathrm{T}} - 1}}$$

$h = 6.546\times10^{-34}$ J-sec is Plank's constant, $k = 1.38\times10^{-23}$ J/$^{\circ}$K is Boltzmann's constant, B is the system bandwidth in Hz, f is the center frequency in Hz, and R is the resistance in ohms.

Noise in Systems (3)

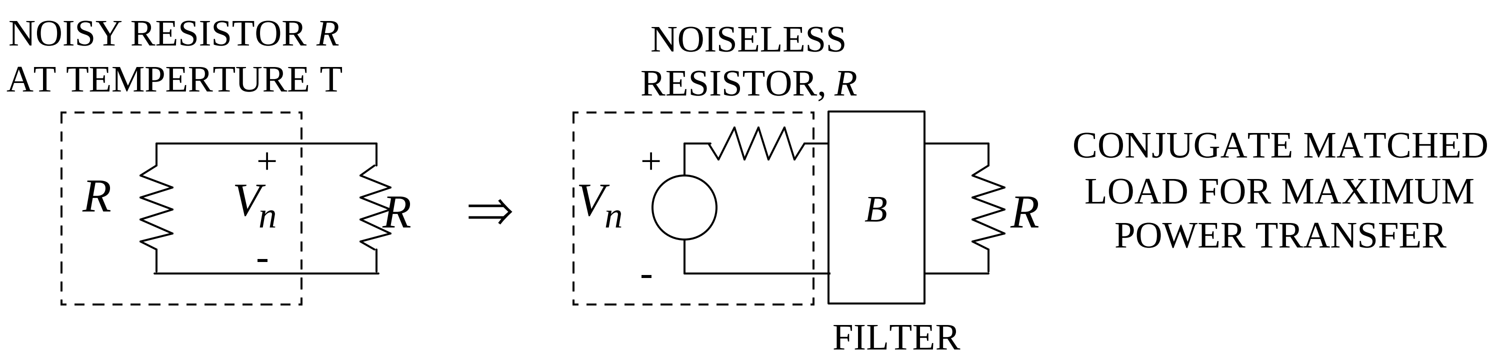
At microwave frequencies $hf << k\mathrm{T}$ and the exponential can be approximated by the first two terms of a Taylor's series

$$e^{hfk\mathrm{T}} - 1 \approx \frac{hf}{k\mathrm{T}}$$

and therefore, $V_n = \sqrt{4k\mathrm{T}BR}$, which is referred to as the Rayleigh-Jeans approximation.

If the noisy resistor is used as a generator and connected to a second load resistor, R, the power delivered to the load in a bandwidth B is

$$N \equiv P_n = \left(\frac{V_n}{2R}\right)^2 R = \frac{V_n^2}{4R} = k\mathrm{T}B \quad \mathrm{W}$$



Noise in Systems (4)

Limiting cases:

- $B \to 0 \quad \Rightarrow \quad P_n \to 0$: Narrow band systems collect less noise
- $\mathrm{T} \to 0 \quad \Rightarrow \quad P_n \to 0$: Cooler devices generate less noise
- $B \to \infty \quad \Rightarrow \quad P_n \to \infty$: Referred to as the ultraviolet catastrophe, it does not occur because noise is not really white over a wide band.

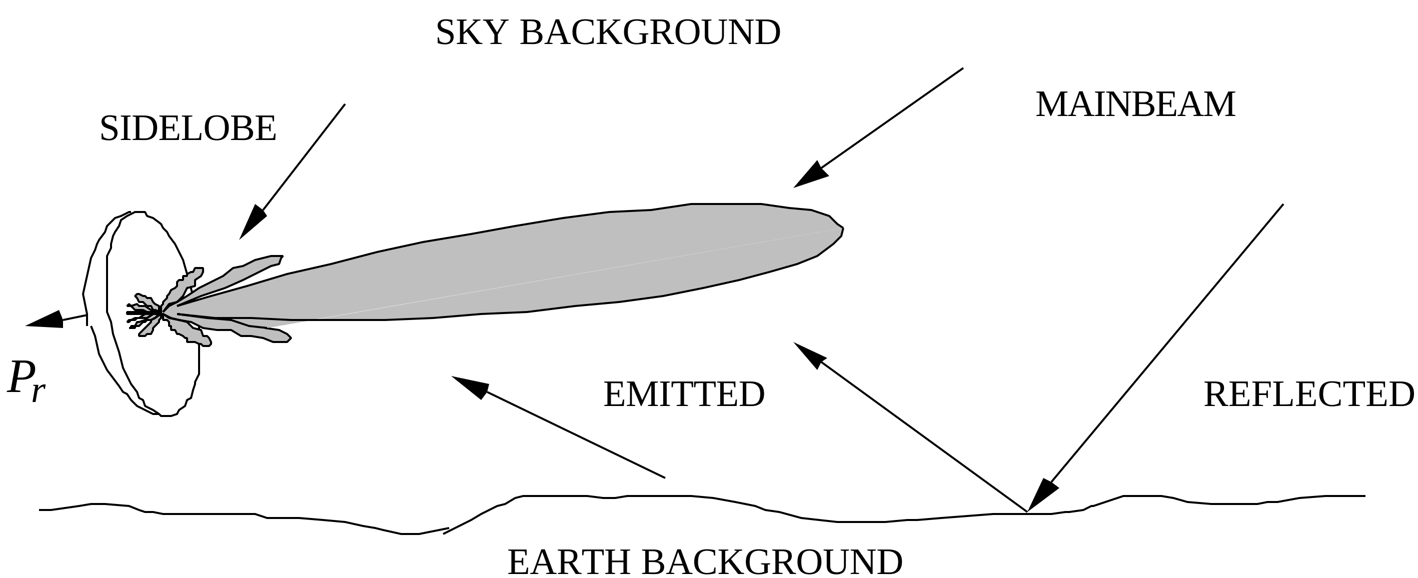
Any source of noise (for example, a mixer or cable) that has a resistance R and delivers noise power P_n can be described by an equivalent noise temperature, T_e. The noise source can be replaced by a noisy resistor of value R at temperature T_e, so that the same noise power is delivered to the load

$$\mathrm{T}_e = \frac{P_n}{kB}$$

The noise at the antenna terminals due to the background is described by an antenna temperature, T_A. The total noise in a bandwidth B is determined from the system noise temperature, $\mathrm{T}_s = \mathrm{T}_e + \mathrm{T}_A$:

$$N = k\mathrm{T}_\mathrm{s} B$$

Calculation of Antenna Temperature



The antenna collects noise power from background sources. The noise level can be characterized by the <u>antenna temperature</u>

$$T_A = \frac{\int_0^{\pi}\int_0^{2\pi} T_B(\theta,\phi)\,G(\theta,\phi)\sin\theta\,d\theta\,d\phi}{\int_0^{\pi}\int_0^{2\pi} G(\theta,\phi)\sin\theta\,d\theta\,d\phi}$$

T_B is the background brightness temperature and G the antenna gain.

Noise in Systems (5)

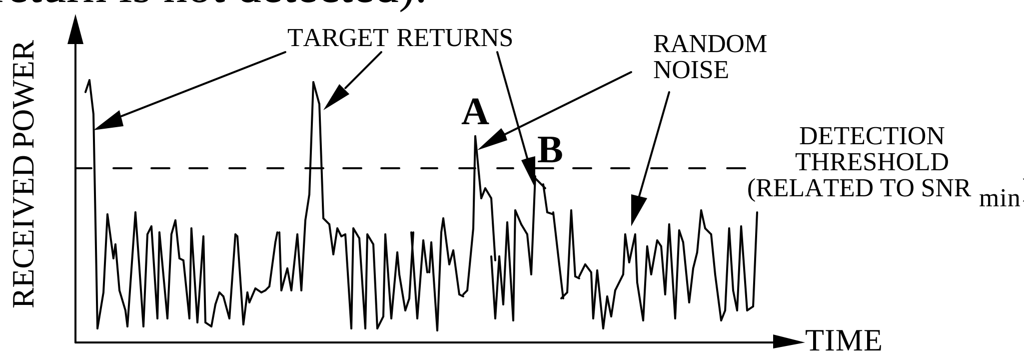
Returning to the radar range equation, we can calculate the signal-to-noise ratio (SNR)

$$\mathrm{SNR} = \frac{P_r}{N} = \frac{P_t G_t s A_{er}}{N(4pR^2)^2} L = \frac{P_t G_t G_r s\, l^2 L}{(4p)^3 R^4 k \mathrm{T}_s B}$$

Given the minimum SNR that is required for detection (detection threshold) it is possible to determine the maximum range at which a target can be “seen” by the radar

$$R_{\max} = \sqrt[4]{\frac{P_t G_t G_r s\, l^2 L}{(4p)^3 (k \mathrm{T}_s B)\,\mathrm{SNR}_{\min}}}$$

If the return is greater than the detection threshold a target is declared. **A** is a false alarm (the noise is greater than the threshold level but there is no target); **B** is a miss (a target is present but the return is not detected).



Noise Figure & Effective Temperature (1)

Noise figure is used as a measure of the noise added by a device. It is defined as:

$$F_n = \frac{(S/N)_{\mathrm{in}}}{(S/N)_{\mathrm{out}}} = \frac{S_{\mathrm{in}} / N_{\mathrm{in}}}{S_{\mathrm{out}} / N_{\mathrm{out}}} = \frac{N_{\mathrm{out}}}{k \mathrm{T}_o B_n G}$$

where $G = \dfrac{S_{\mathrm{out}}}{S_{\mathrm{in}}}$. By convention, noise figure is defined at the standard temperature of $\mathrm{T}_o = 290$ K. The noise out is the amplified noise in plus the noise added by the device

$$F_n = \frac{G N_{\mathrm{in}} + \Delta N}{k \mathrm{T}_o B_n G} = 1 + \frac{\Delta N}{k \mathrm{T}_o B_n G}$$

ΔN can be viewed as originating from an increase in temperature. The effective temperature is

$$F_n = 1 + \frac{k \mathrm{T}_e B_n G}{k \mathrm{T}_o B_n G} = 1 + \frac{\mathrm{T}_e}{\mathrm{T}_o}$$

Solve for effective temperature in terms of noise figure yielding the relationship

$$T_e = (F_n - 1) T_o$$

Noise Figure & Effective Temperature (2)

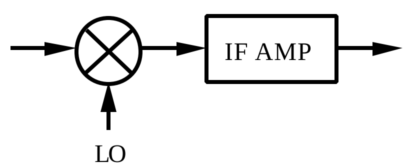
The overall noise figure for M cascaded devices with noise figures $F_1, F_2, \ldots, F_M$ and gains $G_1, G_2, \ldots, G_M$ is

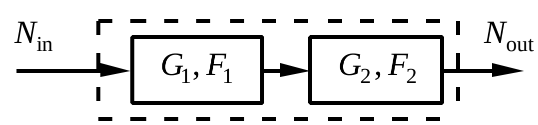
$$F_o = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots + \frac{F_M - 1}{G_1 G_2 \cdots G_{M-1}}$$

The overall effective temperature for M cascaded devices with temperatures $\mathrm{T}_1, \mathrm{T}_2, \ldots, \mathrm{T}_M$ and gains $G_1, G_2, \ldots, G_M$ is

$$\mathrm{T}_e = \mathrm{T}_1 + \frac{\mathrm{T}_2}{G_1} + \frac{\mathrm{T}_3}{G_1 G_2} + \cdots + \frac{\mathrm{T}_M}{G_1 G_2 \cdots G_{M-1}}$$

Example: For the receive channel shown below. The antenna gain is 30 dB, $\mathrm{T}_A = 200$ K; mixer has 10 dB conversion loss and 3 dB noise figure; IF amplifier: 6 dB noise figure



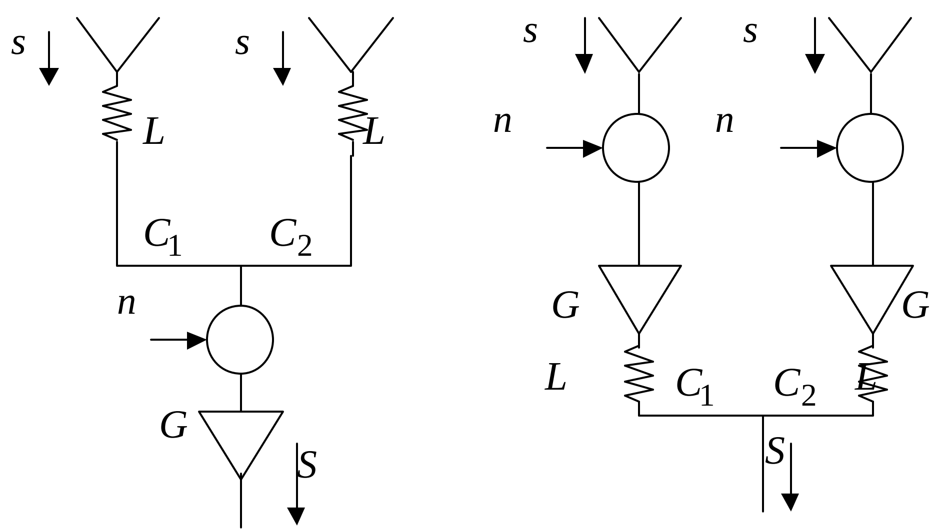


The noise figure of the dashed box is $F_o = F_1 + \dfrac{F_2 - 1}{G_1} = 2 + \dfrac{4 - 1}{0.1} = 32$

Therefore, the system noise temperature is $\mathrm{T}_s = \mathrm{T}_e + \mathrm{T}_A = 31\mathrm{T}_o + \mathrm{T}_A = 9190$ K.

SNR of Active and Passive Antennas (1)

Consider two arrays of isotropic elements: one has the amplifier before the signals are combined and the second has the amplifier after the signals are combined. Note that the latter, amplification after beamforming, is the conventional passive antenna with an amplifier at its output.



Power coupling coefficients for a lossless power divider satisfiy: $C_1 + C_2 = 1$

Passive (left) and active (right) two-element (isotropic) arrays are shown. A uniform plane wave is incident. The signal power *s* is the same at the output of each element. Noise generators are shown in each branch to signify that a white noise power *n* is added to the signal. Each branch has a loss *L*.

We want to compare the signal-to-noise ratio at the outputs for the two cases. (Lower case signifies a “per element” quantity. All quantities are power.)

SNR of Active and Passive Antennas (2)

Since the signal is correlated, voltage is added and then squared to get power, whereas the uncorrelated noise powers are simply added. Assume that the couplers are equal power split, $C_1 = C_2 = 1/2$. For amplification after beamforming:

$$N = nG$$
$$S = \left(\sqrt{sGLC_1} + \sqrt{sGLC_2}\right)^2 = sGL\underbrace{\left(\sqrt{C_1} + \sqrt{C_2}\right)^2}_{=2}$$
$$\frac{S}{N} = \frac{2sLG}{nG} = 2L\left(\frac{s}{n}\right) = (\mathrm{AF})\left(\frac{s}{n}\right)L$$

where AF is the array factor for the two element array and *s/n* is the SNR for a single channel (element). For amplification before beamforming

$$N = nGLC_1 + nGLC_2 = nGL(C_1 + C_2) = nGL$$
$$S = \left(\sqrt{sGLC_1} + \sqrt{sGLC_2}\right)^2 = 2sGL$$
$$\frac{S}{N} = \frac{2sGL}{nGL} = 2\left(\frac{s}{n}\right) = (\mathrm{AF})\left(\frac{s}{n}\right)$$

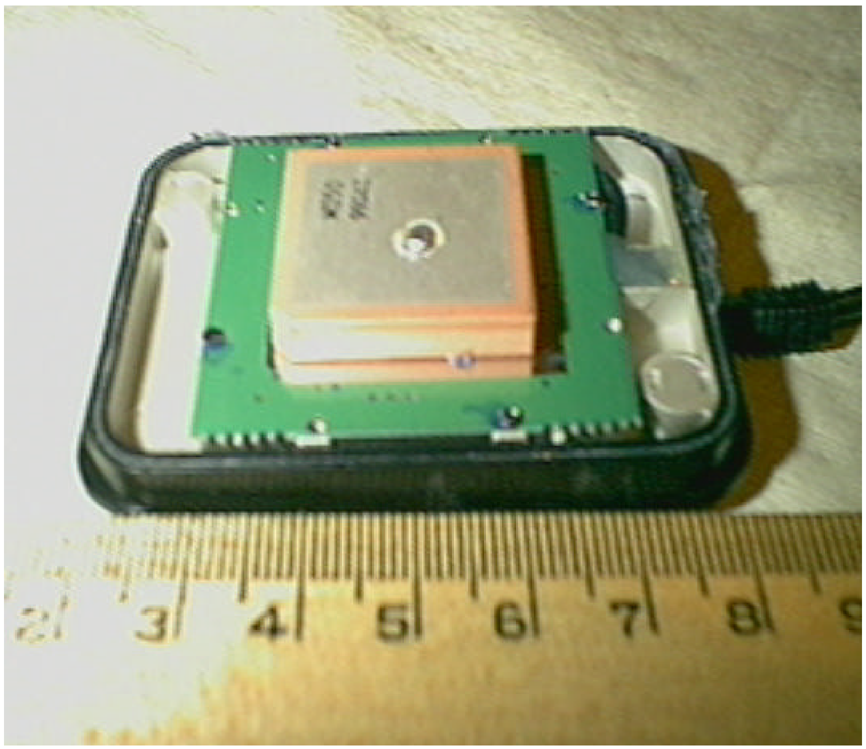
The SNR is not affected by the loss factor *L* when the amplifier is moved to the element. The gain of an active antenna is no longer a measure of the increase in SNR at its output.

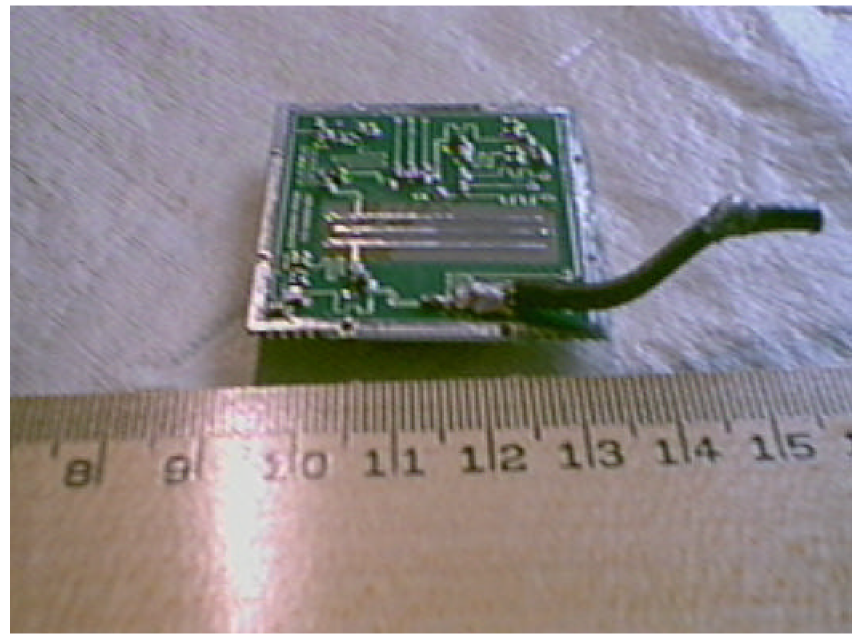
Active GPS Antenna

Active microstrip patch GPS antenna

Top side: radiating patch

Bottom side: amplifier circuit





Comparison of SNR: Active vs Passive

The previous analysis is approximate because it does not take into account the exact relationships between gain and noise figure for each of the devices in a channel. However, it illustrates the important points with regard to amplifiers and beamforming:

PASSIVE ANTENNAS (Amplification after beamforming):

- The antenna gain is the SNR improvement (neglecting noise introduced by the antenna).

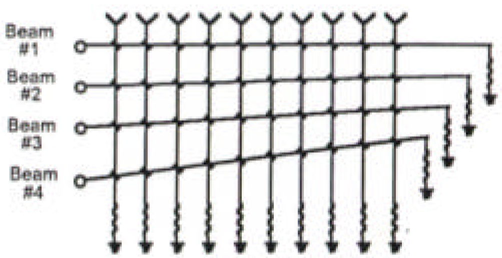
ACTIVE ANTENNAS (Amplification before beamforming):

- The SNR performance can be significantly better than the gain indicates.
- Beam coupling losses can be recovered.
- SNR degradation is determined only by the aperture efficiency. All other losses are recovered.
- The coupler match looking into the sidearms does not affect the SNR.

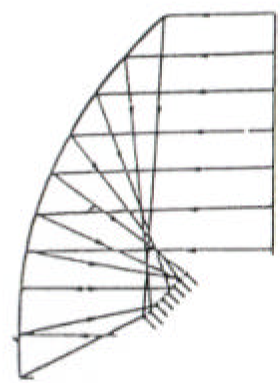
Multiple Beam Antennas

Several beams share a common aperture (i.e., use the same radiating elements)

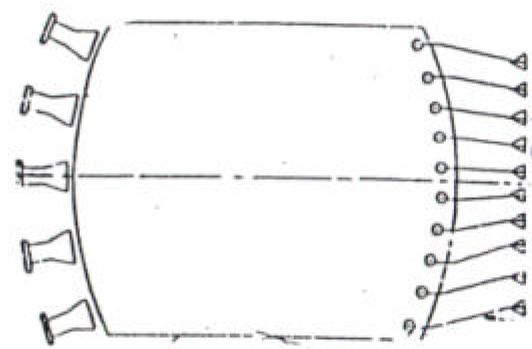
Arrays



Reflectors



Lenses

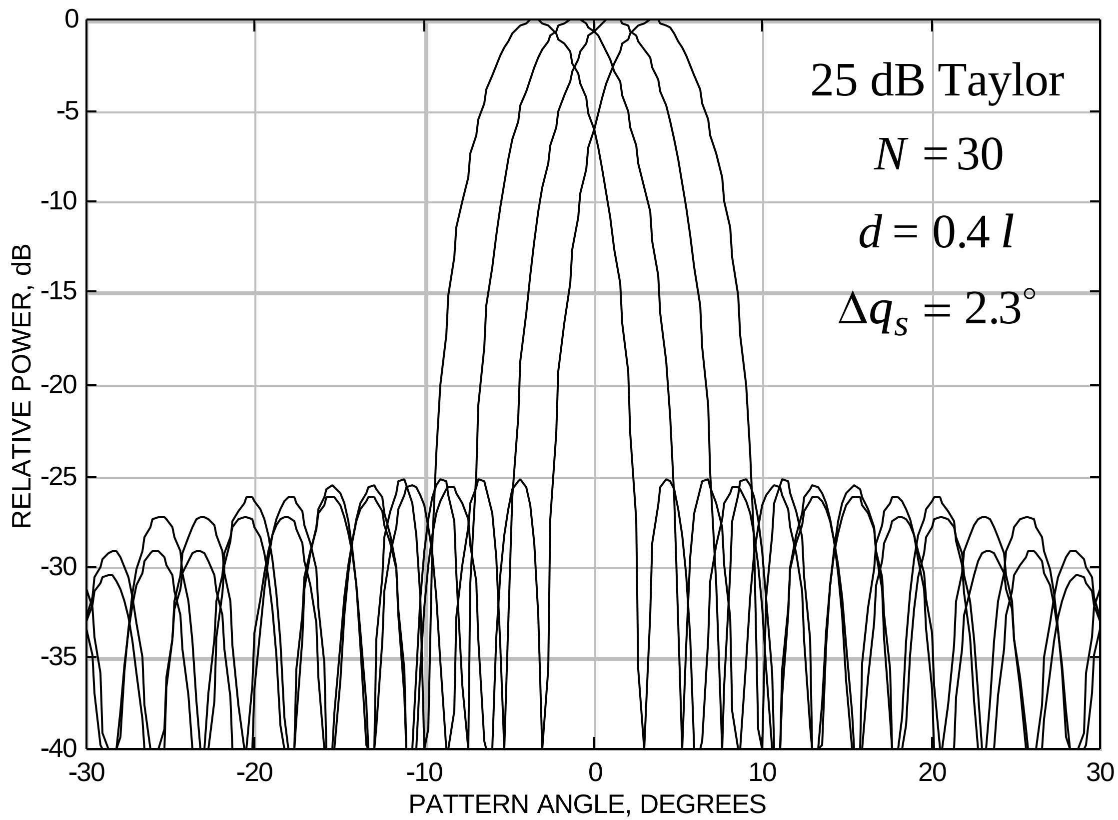


Advantages: Cover large search volumes quickly
Track multiple targets simultaneously
Form "synthetic" beams

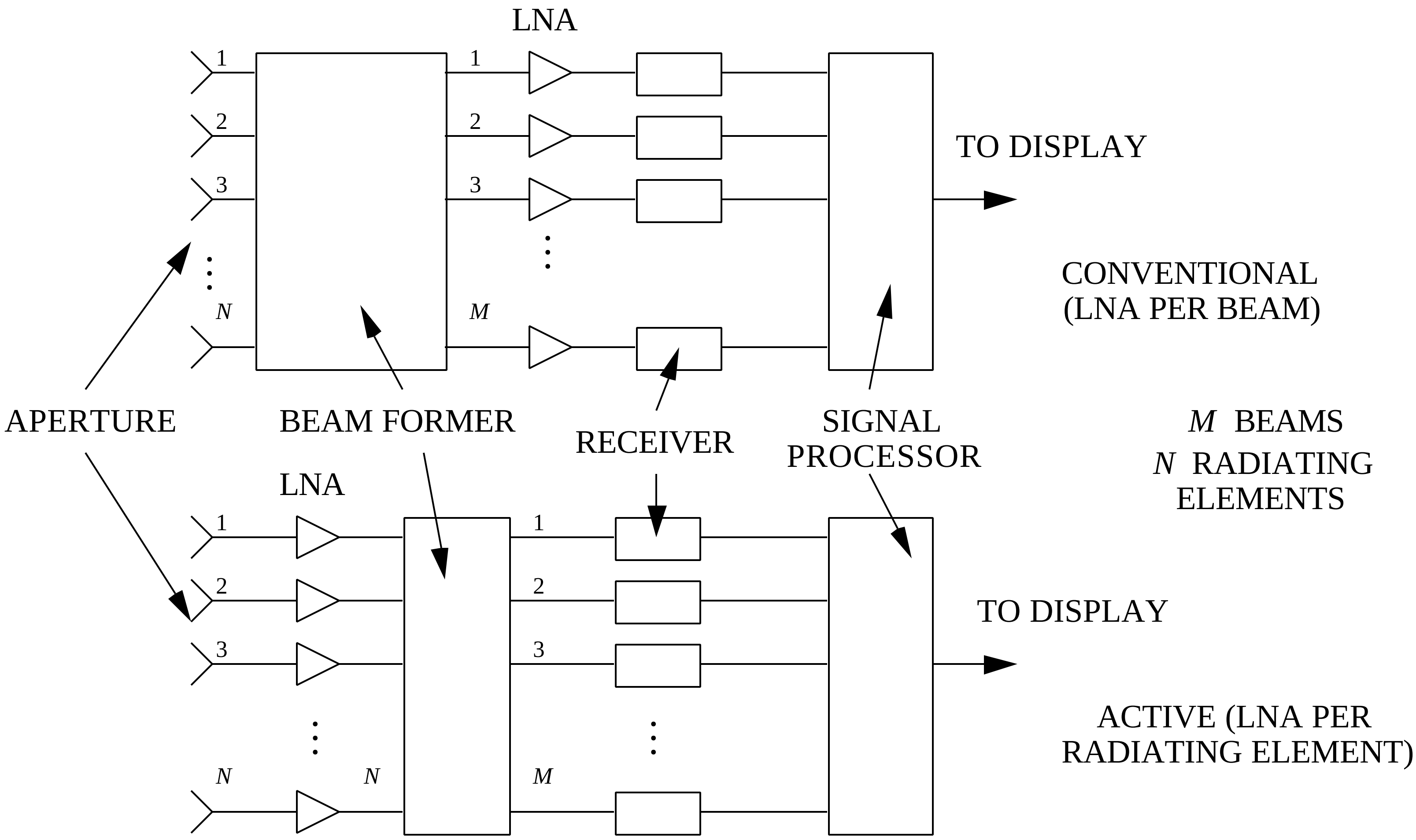
Disadvantages: Beam coupling loss
Increased complexity in the feed network

Sources of beam coupling loss: (1) leakage and coupling of signals in the feed network, and (2) non-orthogonality of the beam patterns

Radiation Patterns of a Multiple Beam Array

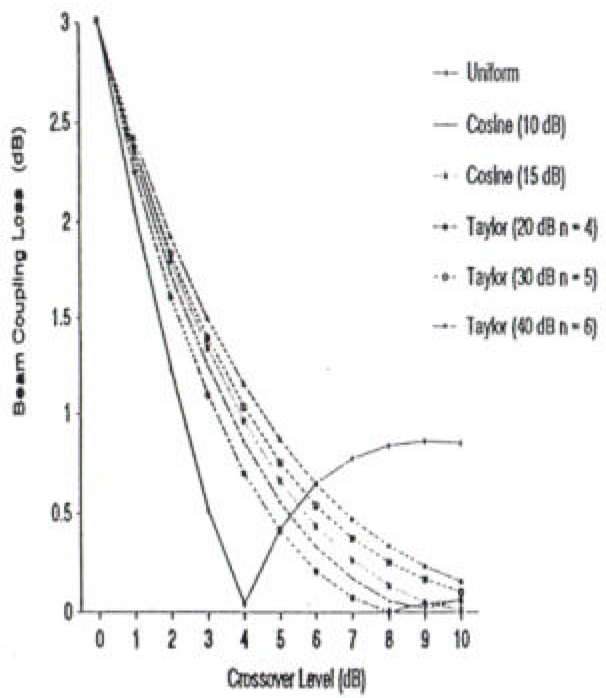


Active vs Passive Multibeam Antennas

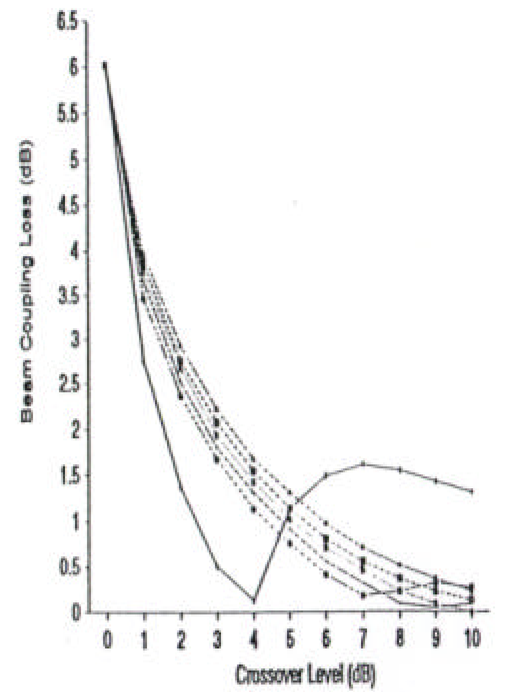


Beam Coupling Losses for a 20 Element Array

2 Beams



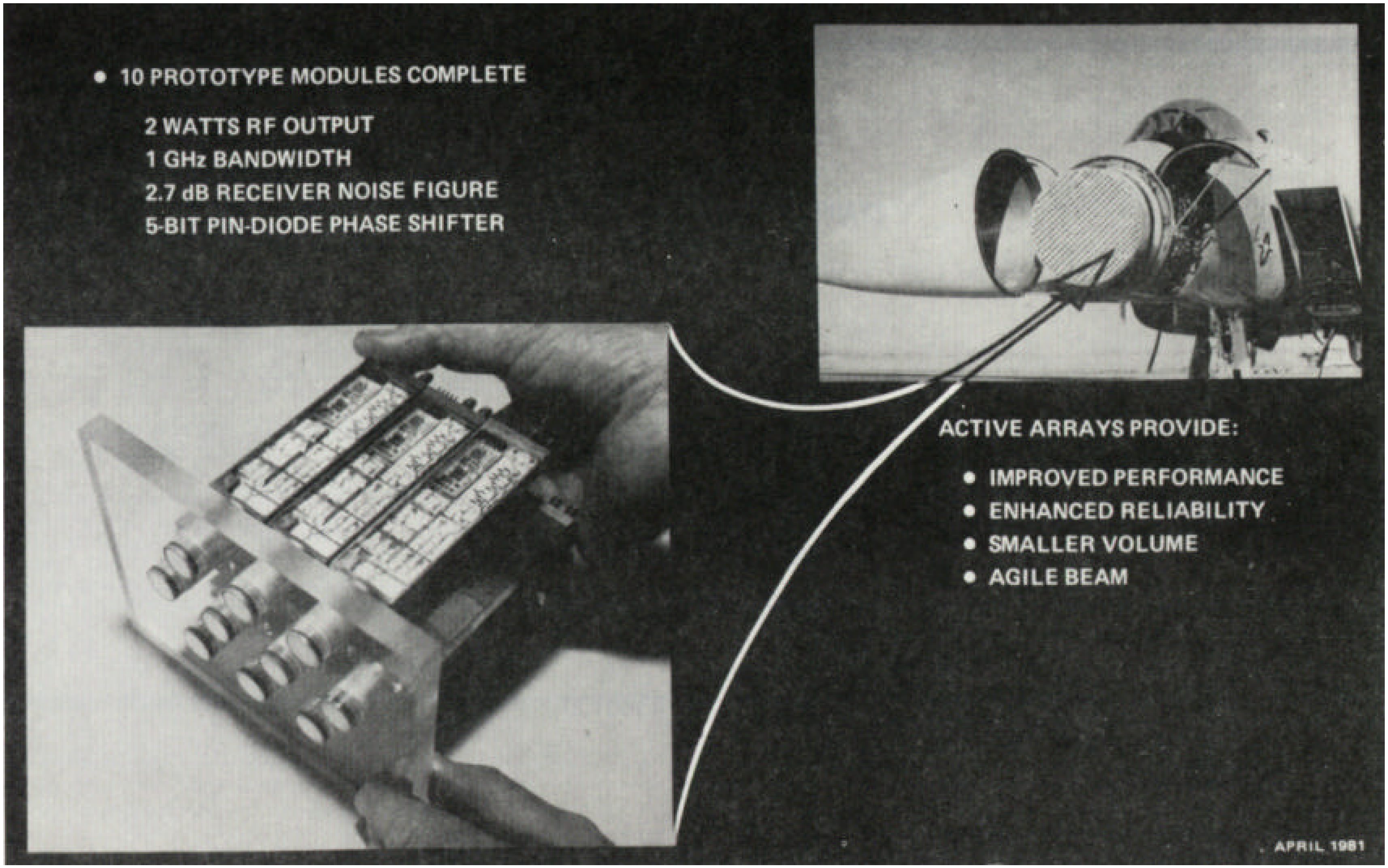
4 Beams



For m and n to be orthogonal beams: $\frac{1}{h}\int_0^{p}\int_0^{2p} \vec{E}_m(q,f)\cdot\vec{E}_n^*(q,f)R^2 \sin q\, dq\, df = 0$

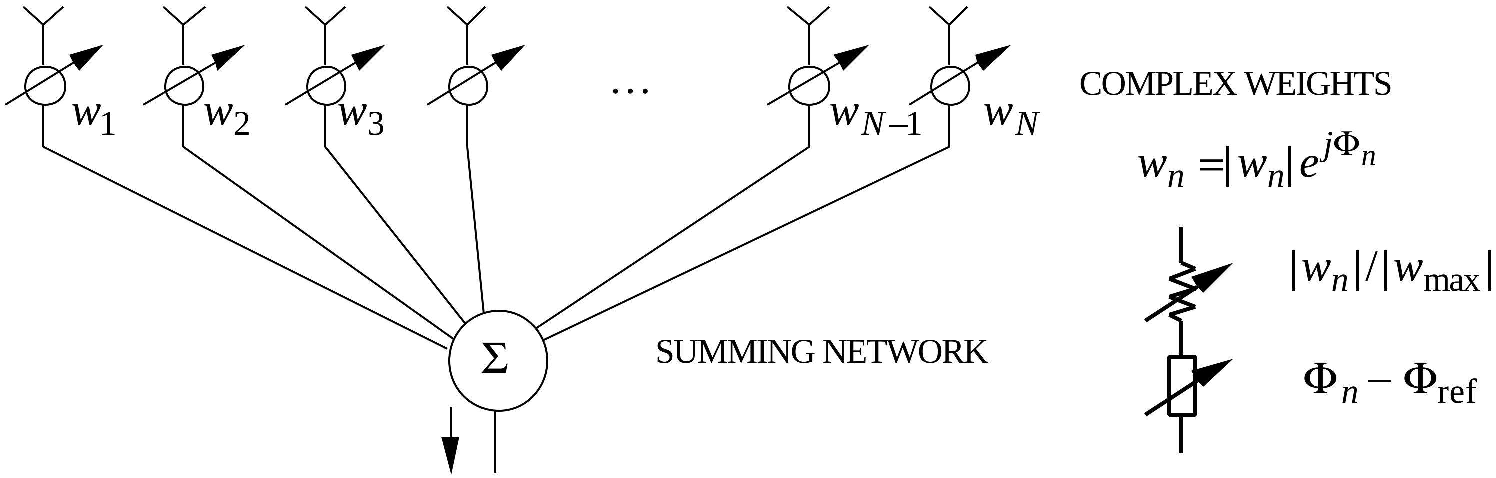
Example: If the beams are from a uniformly illuminated line source then $\vec{E}_n(q,f)$, $\vec{E}_m(q,f) \propto \mathrm{sinc}(\cdot)$ and adjacent beams are orthogonal if the crossover level is 4 dB.

Active Array Radar Transmit/Receive Module



Adaptive Antennas

Adaptive antennas are capable of changing their radiation patterns to place nulls in the direction of jammers or other sources of interference. The antenna pattern is controlled by adjusting the relative magnitudes and phases of the signals to/from the radiating elements.



In principle, a N element array can null up to $N-1$ jammers, but if the number of jammers becomes a significant fraction of the total number of elements, then the pattern degradation is unacceptable.

The performance of an adaptive array depends on the algorithm (procedure used to determine and set the weights). Under most circumstances the antenna gain is lower for an adaptive antenna than for a conventional antenna.

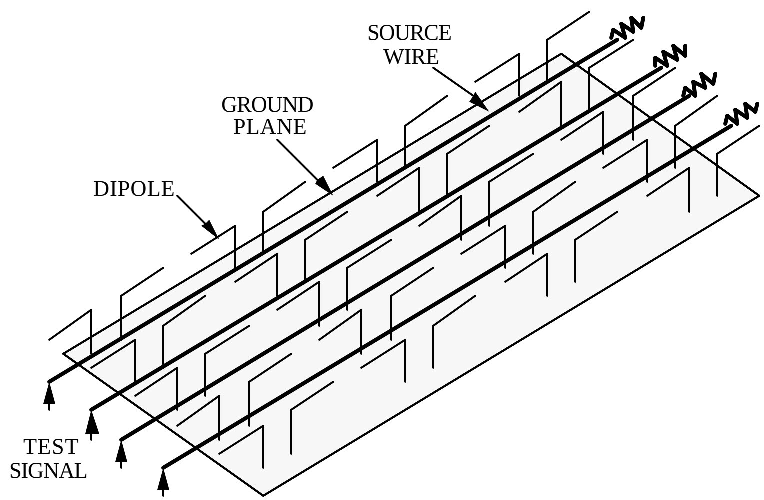
Smart Antennas (1)

Antennas with built-in multi-function capabilities are often called smart antennas. If they are conformal as well, they are known as smart skins. Functions include:

- Self calibrating: adjust for changes in the physical environment (i.e., temperature).
- Self-diagnostic (built-in test, BIT): sense when and where faults or failures have occurred.

Tests can be run continuously (time scheduled with other radar functions) or run periodically. If problems are diagnosed, actions include:

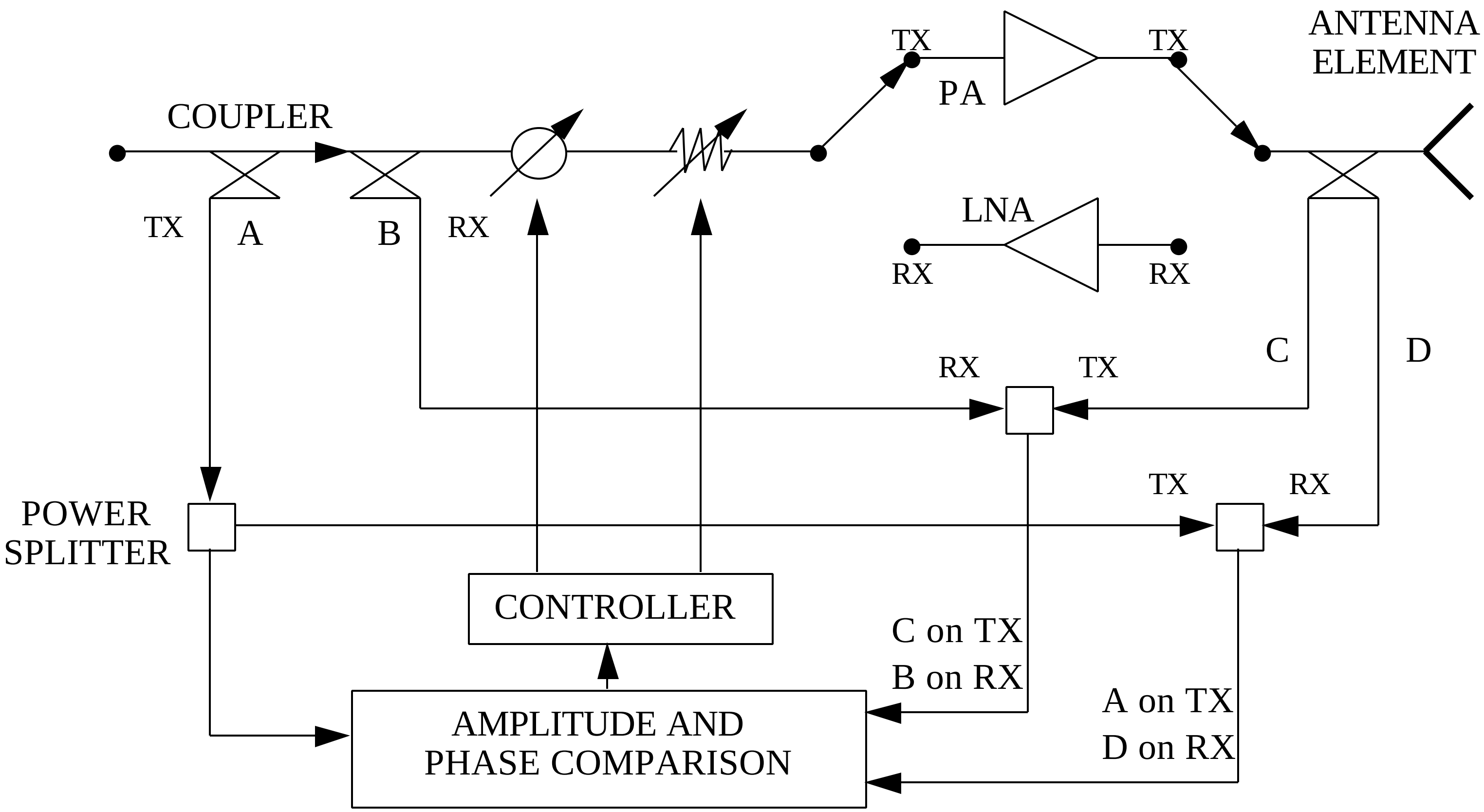
- Limit operation or shutdown the system
- Adapt to new conditions/reconfigure



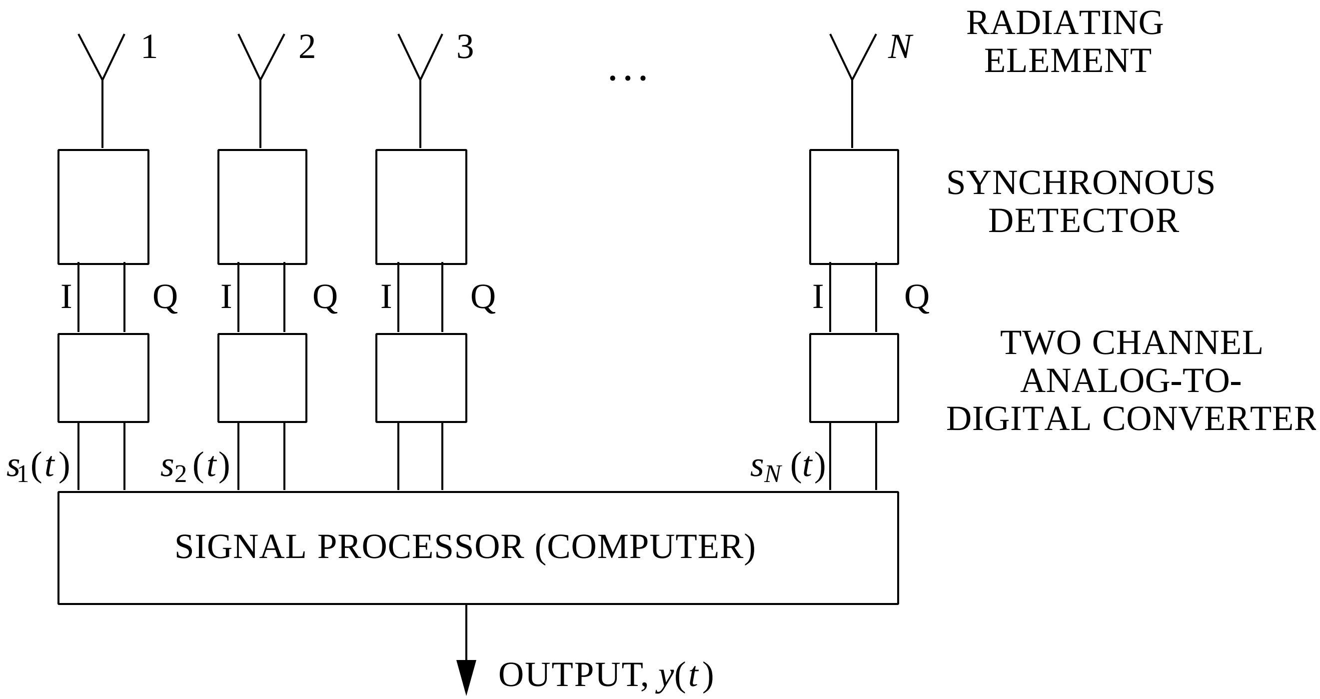
Example: a test signal is used to isolate faulty dipoles and transmission lines

Smart Antennas (2)

Example of a self-calibrating, self-diagnostic transmit/receive module



Digital Beamforming



Assuming a narrowband signal, $y(t) = \sum_{n=1}^{N} w_n s_n(t)$. The complex signal (I and Q, or equivalently, amplitude and phase) are measured and fed to the computer. Element responses become array storage locations in the computer. The weights are added and the sums computed to find the array response. In principle any desired beam characteristic can be achieved, including multiple beams.